Département
D2: Formal Methods

Équipe TYPES

Logic, Proof theory and Programming

Laboratoire lorrain de recherche en informatique et ses applications

Rapport d'activité 2025

En partenariat avec

Contents

Team TYPES

Keywords

Logic, Semantics, Proofs, Refutations, Programs, Resources.

1 Team members, visitors, external collaborators

Faculty Members

- Didier Didier [Team leader UL, Professor]
- Daniel Méry [UL, Assistant Professor]

Research Scientists

- Dominique Larchey-Wendling [CNRS, Researcher]

PhD Students

- Brandon Hornbeck [UL, PhD Student, from October 2021]
- Benjamin Izart [UL - ANR Narco, PhD Student, from December 2022]

Administrative Assistant

- Elsa Maroko [UL]

2 Overall objectives

The TYPES scientific project (Logic, Proof Theory and Programming) consists in studying the links between logic (semantics and proof theory) and reliable system modelling and programming. Our main goal is to study the foundations of new resource models, and their related logics. Reasoning about resources and their evolution is essential to design systems (networks, servers) or programs that access memory and manipulate data structures. Indeed, resources are central in computer science and the concepts of ownership, access, separation, consumption are important for resources. In this perspective, we aim at studying new resource models and logics for system modelling and also new proof structures and calculi dedicated to either automated or interactive theorem proving with a strong focus on proof and refutation construction and on the study of properties like decidability. We aim also at developing works on the formalization and mechanized verification of meta-properties like termination or undecidability. The research activities can be presented following three main and complementary directions: *Resource models and logics* with focus on the modelling of concepts, processes, resources, and complex systems; *Proof structures and calculi* for such logics with focus on structures and proof systems (sequent calculi, proof nets, labelled and label-free structures and calculi) and on adequate proof-search methods; *Formalization and verification of meta-properties* with focus on decidability and undecidability results, in connection with the studied logics.

3 Research program

We organize the research activities in two main themes, one on *Resource models, semantics and expressivity* for modelling complex systems and expressing resource properties and another one on *Proof structures, proof calculi and decision* in order to prove or refute such properties and also to study meta-properties like for instance completeness and decidability. New resource models and logics are motivated by the potential to express high-level resource properties (both qualitative and quantitative), but also by the adequacy between such models and proof-search calculi and procedures. The study of decidable fragments and of new structures, issued from resource constraints, from which validity and countermodel generation can be studied, is a key point. A third complementary theme on *Mechanized verification of meta-theoretic properties* focuses on the formalization in Coq of some meta-properties of logics, mainly undecidability problems.

3.1 Resource models, semantics and expressivity

Reasoning about resources and their evolution is essential to design systems (networks, multicore systems, servers) or programs that access memory and manipulate data structures (with the help of pointers). In this context we study resource models, derived from various interpretations of the composition and decomposition of resources with focus on spatiality and separation (resources, heaps, trees, graphs) and also resource logics in order to express resource properties on data or quantities that can be static (for example about states of memory) and dynamic (for example about program execution). These (abstract and concrete) models and logics are motivated by the expressivity of high-level resource (qualitative and quantitative) properties but also the possible adequacy between such models and some proof calculi. Then the study of decidable logical fragments and the design of new semantic structures, based on particular resource nstraints, are central here.
Our main works and results focus on new resource models and expressivity related to Separation Logic (SL), bunched logics (BI and Boolean BI) and some extensions with action and/or epistemic modalities (DMBI, ESL, ERL) with also a particular interest for their intuitionistic versions.

Our research program in this theme includes different research directions. The main one is focused on the logic of Bunched implications (BI), and its boolean version (Boolean BI), that are logics that allow us to express properties on resources and to provide logical frameworks for the so-called separation logics. The main studies and results concern the extensions of these logics with modalities firstly with standard and secondly with epistemic modalities.
In this context we study a new modal separation logic, DMBI that is Boolean BI extended with two kinds of modalities, allows one to deal with resources having dynamic properties and also to capture some resource evolutions or transformations [20]. We also study extensions of BI with epistemic modalities. A first framework (with D. Pym) is the substructural epistemic logic, ERL, based on Boolean BI, in which the epistemic modalities are parametrized on agents' local resources. The modalities can be seen as generalizations of the usual epistemic modalities. The logic combines Boolean BI's resource semantics with epistemic agency [5] and its expressivity is illustrated with access control problems [23].

Another work is about public annoucement extensions of BI. It starts with a Public Announcement Separation Logic, PASL, (with H. van Ditmarsch) that allows one to consider epistemic possible worlds as resources that can be shared or separated, in the spirit of separation logics [4]. The study focuses on the semantics and the expressivity through examples of modelling systems. Epistemic Separation Logic with Action Models (ESLAM), that can be seen as a generalization of Public Announcement Separation Logic (PASL). We generalize the dynamic aspects of PASL, by defining ESLAM, in which we replace public announcements with action models, motivated by their ability to model factual change, and instances of a more nuanced, private communication [31].
Our aim is to go on to explore modal/epistemic separation logics in order to study the general question "How can we combine knowledge and resources?" in the direction of modelling information as a resource. The design of new resources models and resource logics in this perspective is a challenge.

A recent research direction is related to the ANR project NARCO "Non-Aggregative Resource COmpositions" that is focused on the development of a methodology for modeling and verification of modular systems built from resources composed using non-aggregative operators. We need to define new logical languages for reasoning about resources in a distributed component-based system. These resources can be passive or active and require rich semantic domains to model their interactions and composition. In particular, the composition will be typically modeled using non-aggregative operations, which poses non-trivial questions relative to the logical properties of the connectives used to represent composition. In this context we aim at studying two main points: a semantic theory of (non-)aggregative composition, based on identifying the axiomatic properties of aggregation, establishing formal relations between these properties and illustrating this theory by modeling existing systems; resource logics for reasoning about the dynamic updates of the system in a local/compositional fashion.

Concerning our study of various semantics we want to explore the interest of a new class of models, called Topological Beth Semantics, for various logics like, for instance, extensions of BI logic. We can start from our works for ISCI where these models which can be viewed as algebraic counterparts (and extensions) of sheaf-theoretic topological models of intuitionistic logic [6].

3.2 Proof structures, proof calculi and decision

We develop new calculi and to propose in some cases decision procedures. For that we need to build particular proof structures and calculi and such a design is a real challenge. We have, for instance, to capture inside the logics interactions between separation and modalities with specific semantic constraints and structures (resource graphs), to design proof calculi that generate proofs (certification) and counter-models (failure analysis), but also to solve decidability and undecidability problems through proof-search.
Our works and results concern the logics previously mentioned, namely SL, BI and BBI variants or modal and epistemic extensions of bunched logics, but also intutionistic versions of other resource logics. Then we have defined various proof structures like labelled sequents, label-free sequents, tree-sequents, resource graphs that appear necessary to solve some important proof-theoretical questions.

For the new modal separation logics [20, 21], extensions of BI, a sound and complete labelled tableau calculus with constraints, is defined. The resource semantics of each logic is captured through specific labels and constraints in the corresponding calculus that deals with specific labelled sequents. Even if proving the soundness is standard, proving completeness is a real challenge for each logic depending on the modalities. For the epistemic separation logics [4, 5] the same approach, adapted to the specificities of the logic, leads also to a sound and complete labelled tableau calculus with constraints with counter-model extraction.

The study of proof translations between labelled calculi for BI and the label-free calculus LBI is a recent topic. In this context we first define a labelled sequent calculus, called GBI, in which labels and constraints reflect the properties of a specifically tailored Kripke resource semantics of BI. Then we show how to translate any LBI-proof into a GBI-proof. From this translation we devise a property such that any GBI-proof with this property can systematically be translated to an LBI-proof [24]. Our aim is to study such a transformation but without using this property. It is a real challenge and the solution will allow us to analyse relationships between the different semantics of BI logic annd to solve an open question related to the completeness of some semantics.

In the context of the ANR project NARCO we aim at studying and developing calculi for the new non-aggregative logics investigated in the research directions about resource models and resource logics. We want to explore two kinds of calculi: internal calculi (mainly label-free calculi) and external calculi (mainly labelled deduction systems). Therefore, studying relationships and proof transformations between both types of calculi is another important concern of this task. We first study non-aggregative composition of resources and design of calculi from our previous works on BI and BBI logics.

In addition to these works on bunched and separation logics we also investigate intuitionistic versions of some logics for which calculi and proof-search methods are missing. For multiplicative intuitionistic linear logic (MILL) we want to propose new proof systems starting from our recent a connection-based characterization which is based on labels and constraints and proved sound and complete for the logic [25]. In this case we can derive a labelled sequent calculus for MILL. Our aim is now to provide a new connection method based on a refinement of this characterization with an algorithm for the resolution of constraints between labels.

The new class of models, called Topological Beth semantics for ISCI, allows us to derive labelled sequent calculi with good properties (adequacy with Hilbert axiomatization, cut-elimination). Using

a key regularity property of the forcing relation in Beth models, we show through different steps the decidability of ISCI [6].
Our goal in this direction is to refine and complete this work and to study in detail proof transformations between two main sequent calculi: G3ISCI that is a label-free mono-conclusioned sequent calculus proven sound and complete w.r.t. the class of Kripke models, and L1ISCI that is a labelled multi-conclusioned sequent calculus proven sound and complete w.r.t. the class of recently introduced (Topological) Beth models. One goal is also is to generalize this approach based on this specific semantics to other logics like extensions of BI logic with modalities.

3.3 Mechanized verification of meta-theoretic properties

This research axis is about the mechanization of proofs and the certification of algorithms with the orof assistant Coq. It leads to different development projects of two kinds:
- the mechanization of new results or results only presented in articles, like the elementary models of BI or the extraction of the generalization of the F91 function.
- the revision of previous mechanizations under the form of “proof pearl” that allows us to communicate ideas through source code in Coq.
For instance a recent result on the extraction of μ-recursive functions that clarifies the approach of 2017 by using the Braga method [10]. The approach by “pearl” is motivated by an increasing interaction with developers using tools for formal proofs like Coq. It allows to learn concepts and methods through the provided code that is readable and simple.

In the past years, we have mainly developed tools for the mechanised and constructive verification in Coq of meta-theoretic properties of logical systems, i.e., cut-elimination, decidability or undecidability results. Such works involve dealing with computability in a formal way, and thus a new method called "Braga method" is proposed to implement and certify complex recursive algorithms in Coq, via extraction. This includes in particular nested and mutually recursive schemes, but also partially terminating algorithms [10].

In relation with the other research axes we focus on decidability and undecidability results and also on termination. For instance We propose the first fully constructive (and also mechanised) proof of the decidability of implicational relevance logic, converting Kripke and Curry’s argumentation into a constructive proof in Coq [26, 8]. In order to tackle undecidability results, we study computability theory in Coq relating the formal concept of "μ-recursive function" (constants, successor, projection, composition, primitive recursion, minimization) with Coq’s internal notion of computability. A first result consists in proving that any recursive function of which the totality can be proved in Coq can be represented by a Coq definable function of this type [28].

A work (with Y. Forster) provides a mechanized proof of undecidability for entailment in intuitionistic linear logic, by a chain of reductions from Post’s correspondence problem (PCP)[22]. In this context another work (with Y. Forster) contributes to the first fully mechanized (and moreover constructive) proof of the DPRM theorem which allows to establish the undecidability of Hilbert’s tenth problem, i.e., the solvability of Diophantine equations, a critical new seed of undecidability [29].

A recent work study the undecidability of an extension of multiplicative and exponential linear logic (MELL), mechanised in Coq. The IMSELL fragment is intuitionistic (single conclusion) and extends MELL by adding two modalities in addition to the exponential. The proof deals with the mechanization of the undecidability of the halting problem for two-counters Minsky machines (MM2) and the completeness of the reduction from MM2 to IMSELL proceeds via a short semantic argument, based on trivial phase semantics [9].

Moreover a method is proposed in order to encode termination predicates for recursive algorithms into bar/accessibility predicates which allowed to use those predicates as means to define those algorithms in Coq so that they could be extracted as is. This framework also allows to prove correctness and characterize the domain of termination. This leads to the extraction of different kinds of algorithms [30, 27].

In collaboration with Y. Forster and the team of G. Smolka at Saarbrucken, we can also mention the development of the "Coq Library of Undecidability Proofs" (https://github.com/uds-psl/coq-library-undecidability) which is a synthetic framework to mechanically verify many-one constructive ctions, establishing undecidability results in Coq. The library now contains more than 50 undecidable problems and reductions between them (see [21]).

4 New softwares, platforms, open data

B. Hornbeck and D. Méry have recently developed an automated theorem prover, called AutoPSI, for the intuitionistic non-Fregean sentential calculus with Suszko's identity ISCI. The concepts and foundations on which the prover design is based, were first presented in [6]. The ISCI theorem prover is implemented with different proof search strategies and procedures and we have developed tests and benchmarks for the logic ISCI but also for the restricted case of intuitionistic logic [16]. This prover is still under development, taking into account the current refinements in the proof-search process.

D. Larchey-Wendling has a strong activity about the development of formalizations and verification in Coq of various meta-properties of logics (decidability or undecidability) and of important theorems. We can mention the strong normalisation for System F, the extraction and proof of termination of the generalization of the function F91 de McCarthy, and also the formalization of the second theorem of recursion (Kleene) or a functional language of first-order dealing with binary trees with extraction of a certified Quine towards Ocaml. These works are on GitHub or GitLab but some are not public for the moment. D. Larchey-Wendling has also developed (with J.F. Monin) in Coq the extraction of μ-recursives functions that is a fundamental model of computation of the calculability theory [19].
D. Larchey-Wendling participates also to the development of a library of formalized undecidable problems in Coq (with MPI and the University of Saarbrucken) [21].
Recently he has developed `Coq-Kruskal` that is a Coq library for Almost Full (AF) relations. It a set of tools to manipulate AF relations (i.e. constructive WQOs) in Coq, including high-level theorems such a Dickson's lemma, Higman's lemma et theorem, and Kruskal's tree theorem. A summary with links can be found here: `https://github.com/DmxLarchey/Coq-Kruskal`. It is distributed and maintained within the opam archive for Coq. We can mention two direct applications of the `Coq-Kruskal` project that are Harvey Friedman's TREE(n) monster function in Coq and the Karp-Miller algorithm for the covering problem in Coq.

5 New results

5.1 Resource models, semantics and expressivity

Participants: Didier Galmiche, Brandon Hornbeck, Benjamin Izart, Dominique Larchey-Wendling, Daniel Méry.

We consider the general question "How can we combine knowledge and resources?". For that we can go in the direction of modelling uncertainty about resources [4]. But one can also go in the direction of modelling information as a resource. We very clearly go in that, novel, direction but incoming information is highly dependent on context and may have side effects, so it is difficult to separate/decompose, which goes against the grain of separation logics. But it is therefore a challenge we propose to meet.

We present an extension of (bunched) separation logic, Boolean BI (BBI), with epistemic and dynamic epistemic modalities. This logic, called Action Model Separation Logic (AMSL), can be seen as a generalization of Public Announcement Separation Logic (PASL) [4] in which we replace public announcements with action models. Then we not only model public information change (public announcements) but also non-public forms of information change, such as private announcements. In this context the semantics for the connectives from separation logic are epistemic versions of their usual semantics. This is because formulas are interpreted in states, not in resources, and agents may be uncertain between different states representing the same resource. We present the logic AMSL and its semantics, with a detailed case study that highlights its interest for modeling. We also prove the elimination of the dynamics modalities and discuss some alternative epistemic semantics for the separation connectives [2].

In this context we also propose a separation logic where resources are histories (sequences) of epistemic actions so that resource update means concatenation of histories and resource decomposition means splitting of histories. This separation logic, called AMHSL, allows us to reason about the past: does what is true now depend on what was true in the past, before certain actions were executed? We show that the multiplicative connectives can be eliminated from a logical language with also epistemic and action model modalities, if the horizon of epistemic actions is bounded [14]. We also define a variant of this epistemic logic with separation that only requires a single multiplicative implication in the language. A journal paper including all these results has been submitted in 2025.

We study an extension of BI logic, called BiBI with a new connective called co-implication, in the spirit of Bi-intuitionistic (BiInt) logic. In this context we propose new results for BI logic and also for the BiIint logic. For BI logic, for which a bunched sequent calculus (LBI) has been initially defined we define a topological Beth (TB) semantics and propose an axiomatization through a Hilbert-style calculus (HBI). We prove a soundness result w.r.t. LBI, namely if a formula is provable in HBI it is provable in LBI, and a completeness result w.r.t. the TB semantics, namely if a formula is valid in this semantics then it is provable in HBI. For BiInt logic we consider a generalized Hilbert calculus (GHBiInt) and define also a TB semantics. Then we prove the soundness and the completeness of the calculus w.r.t. such a semantics. Back to BiBI logic we define, from the previous results, a Hilbert calculus and also a topological Beth (TB) semantics and prove the soundness and completeness of the calculus w.r.t. this semantics. All these new results about axiomatizations and semantics have been submitted in 2025 for publication.

There are very many approaches to systems modelling, typically building on complex structural and dynamic frameworks. For example, there are simulation modelling tools based on a theoretical treatment of a ‘distributed systems metaphor’ for system modelling. These tools are based on the concepts of location, resource, and process. Recently we have explored a foundational modelling framework based on minimal assumptions, starting from a primitive notion of behaviour, and we have shown how such an approach allows us the recovery of the key ideas, including a generalized CAP theorem, required for effective modelling of and reasoning about ecosystems of systems. We establish a logic of behaviours and use it to express local reasoning principles for the compositional structure of systems [17].

We also introduce Bifurcation Logic (BL), which combines a basic classical modality with separating conjunction, $*$, together with its naturally associated multiplicative implication, $-*$, that is defined using the modal ordering [13]. Specifically, a formula $\phi_1 * \phi_2$ is true at a world w if and only if each ϕ_i holds at worlds w_i that are each above w, on separate branches of the order, and have no common upper bound. In this context we define a relational semantics for BL/ Moreover we illustrate the use of BL through an example of modelling multi-agent access control that is quite generic in its form, suggesting many applications.

5.2 Proof structures, proof calculi and decision

Participants: Didier Galmiche, Brandon Hornbeck, Benjamin Izart, Dominique Larchey-Wendling, Daniel Méry.

In the continuation of previous works [25] we study how to design proof systems for multiplicative intuitionistic linear logic (MILL). In this perspective we have defined a new connection characterization for MILL, based on important refinements of our previous characterization [25], and the related connection method based on the definition of an algorithm for the resolution of constraints between labels. A submission of these results is expected in the beginning of 2026.

We study the Intuitionistic non-Fregean Sentential Calculus with Identity (ISCI) for which we recently obtained new results on semantics (a new Topological Beth semantics) and on proof theory (new labelled calculi with good properties) that lead us to prove the decidability of the logic [6].
We have completed and refined this work with complementary results and new arguments about the decidability result and also with a discussion about the proof search method and its implementation with one of the labelled calculi [1]. Moreover a theorem prover for ISCI is implemented with different proof search strategies and procedures and also tests and benchmarks [16].

In a recent work on ISCI we study proof translations between two main sequent calculi, in both directions. One, called G3-ISCI, that is a label-free mono-conclusioned sequent calculus proved sound and complete w.r.t. the class of Kripke models, and another one, called L1ISCI, that is a labelled multi-conclusioned sequent calculus proved sound and complete w.r.t. the class of recently introduced (topological) Beth models [6]. The first result is the translation from G3-ISCI-proofs into L1ISCI-proofs that is proved sound. But the main challenge is the translation from L1ISCI-proofs into G3-ISCI-proofs, as going from a labelled multi-conclusioned to a label-free mono-conclusioned sequent calculus is known to be a non-trivial problem in general. Our approach to solve it proceeds in several intermediate steps that involve the definition of a new nested sequent calculus for ISCI, called NS-ISCI, and its linear variant LNS-ISCI, both of which being proven sound and complete. These new results about calculi and proof transformations have been submitted in 2025 for publication in a journal.

In the continuation of the previous French ANR - Austrian FWF project, called Ticamore, we have studied a broad account of the various sequent-based proof formalisms in the proof-theoretic literature, focusing on various modal and tense logics, intuitionistic logic, conditional logics, and bunched logics. First we show how these sequent-based formalisms can be placed in a hierarchy in terms of the underlying data structure of the sequents. We then discuss how this hierarchy can be traversed using translations. Translating proofs up this hierarchy is found to be relatively straightforward while translating proofs down the hierarchy is substantially more difficult. Finally, we inspect the prevalent distinction in structural proof theory between ‘internal calculi’ and ‘external calculi’ and discuss the ambiguities involved in the informal definitions of these categories, and we critically assess the properties that (calculi from) these classes are purported to possess [12].

In the context of our general study of extensions of Bunched Implications logic (BI) and separation logics, with a strong focus on the links between semantics, calculi and proofs [15], we study a new extension with temporal modalities that appears to be a real challenge compared to our previous works [20, 21]. We define the logic of Linear Temporal Bunched Implications (LTBI), a temporal extension of the Bunched Implications logic BI that deals with resource evolution over time, by combining the BI separation connectives and the LTL temporal connectives. We first present the syntax and semantics of LTBI and illustrate its expressiveness with a significant example. Then we introduce a tableau calculus with labels and constraints, called TLTBI, and prove its soundness w.r.t. the Kripke-style semantics of LTBI. Finally we discuss and analyze the issues that make the completeness of the calculus not trivial in the general case of unbounded timelines and explain how to solve the issues in the more restricted case of bounded timelines [3]. We have recently solved the general case by defining a new labelled sequent calculus for LTBI and also cyclic proofs in this logic, in order to fix the completeness result. These new results about completeness for LTBI have been submitted in 2025 for publication.

We also study the extension of BI logic, called BiBI, with a new connective called co-implication initially studied in Bi-intuitionistic logic. From both Kripke semantics and a new topological Beth semantics for BiBI we define labelled calculi with labels capturing these semantics but with a strong interest for the labelled calculus derived from TB semantics, mainly because of its regularity property that allows a specific management of labels. Then we prove the soundness and completeness of the calculi w.r.t. both semantics. The new results about TB semantics and the labelled calculus for BiBI, that is proved sound and complete w.r.t. this semantics have been submitted in 2025 for publication.

The two previous works on LTBI and BiBI logics can be related to the study of resource compositions that are non-aggregative. In this context and in the perspective of the ANR project NARCO “Non-Aggregative Resource COmpositions” we start to study how existing calculi, for instance for BI and its extensions, can be adapted and refined for dealing with non-aggregative composition and then non-aggregative logics.

After the study of Bifurcation Logic (BL) from both semantic and modelling points of view we provide a labelled tableaux calculus for BL and establish soundness and completeness relative to its relational semantics. The standard finite model property fails for BL. However, we show that, in the absence of $-\!\!*$, but in the presence of $*$, every model has an equivalent finite representation and that this is sufficient to obtain decidability [13].

5.3 Mechanized verification of meta-theoretic properties

Participants: Dominique Larchey-Wendling.

We study the formalization of elementary models (ordered partial monoids) of the BI logic. The elementary models, namely those generated by the proof-search in the labelled tableaux calculi, are partially ordered monoids, with squares, and regular. It is on Github but not public for the moment .
We also study the strong normalisation for System F, with a "proof pearl" that gives a constructive proof of the strong normalisation of system F. It simplifies and revised a previous work. It is available on GitHub: `https://github.com/DmxLarchey/SystemF`.

We also propose "Hercules kills the Hydra in less that 300 lines of Coq." that is a "proof pearl" in Coq of the victory of Hercules against the Hydra. It uses a short proof of the wellfoundness of the order nested on multi-sets of n-ary trees. It is available on GitHub: `https://github.com/DmxLarchey/Hydra`.

We study (with J.F. Monin) the extraction of μ-recursives functions in Coq by showing that Coq can represent in functional form, extractable in OCaml, all the μ-recursive algorithmes, that is a fundamental model of computation of the calculability theory [19]. We also study the formalization of the second theorem of recursion (Kleene) or a functional language of first-order dealing with binary trees with extraction of a certified Quine towards Ocaml. The internal représentation of the heap is central w.r.t. the complexity. The result is on GitLab but not public for the moment.

We give a formalization in Coq of the result of the paper "Well Founded Unions" from Dawson et. al (IJCAR 2018) by using a constructive approach based on the inductive predicate Accessibility. The original proof in Isabelle is based on decreasing sequences and then needs the excluded middle and the axiom of choice. This work emphasizes the interest of the technique "union of wellfounded relations" to study nested orders on n-ary trees. The result is on GitLab but not public for the moment.

We extract and published sub-libraries on opam/Coq : KruskalTrees and KruskalFinite that can be found at `https://github.com/DmxLarchey/Kruskal-Trees` and `https://github.com/DmxLarchey/Kruskal-Finite`. They are extracted form the proof of Kruskal theorem in order to allow their use in external projects. We also consider the mechanization in Coq of the algorithm that computes covers on Petri nets (Karp-Miller) from the library Almost-Full linked to the Kruskall theorem. We also study the extraction and proof of termination of the generalization of the function F91 de McCarthy. The result is on Github but not public. In this context we also design a constructive tool for transferring the AF property between relations [18].

A recent work presents alternatives for (the contrapositive of) König (infinity) lemma that can be used as low-cost replacements in constructive contexts. They are illustrated with two example applications. Several variants of (the contrapositive of) König's lemma, also known as the "FAN theorem" in constructive mathematics, are given axiom free proofs in Rocq. The work is based on an original proof of an abstracted version of the result grounded on the notion of inductive cover as used in inductive topology (see e.g. Sambin, Coquand, Valentini). It has been adapted to inductive bars and almost full relations and is suitable for the replacement of König's lemma in the context of the constructivisation of classical proofs, when possible. These results have been published in 2025 in the Postproceedings of TYPES 2024 in the paper *Constructive substitutes for König's lemma* (https://doi.org/10.4230/LIPIcs.TYPES.2024).

In constructive commutative algebra, we revive the bar inductive characterization of Noetherian rings. We contribute the first constructive (axiom free) implementation of Hilbert's basis theorem, in the Rocq proof assistant, following the paper outline given in Coquand&Persson (TYPES 98). We also propose a genuine proof that (bar) Noetherian rings are closed under direct products, constructively and without assuming further assumptions like e.g. strong discreteness. These results have been submitted in 2025 and accepted for publication in 2026 at CPP 2026 (Certified Programs and Proofs) Conference.

6 Partnerships and cooperations

6.1 European initiatives

We have a strong collaboration with the Programming Principles, Logic, and Verification (PPLV) Group (Prof. D. Pym) at University College London (UCL), UK, with works on semantics, separation logics with modalities, and formal modelling of distributed systems.

We (D. Galmiche and D. Méry) have visited the UCL group in London in November 2025 (3 days) in order to work on "Multi-agent Bifurcation Logic" and to fix other research directions for this collaboration.

We have also a collaboration with the Logic Group (of Prof. A. Ciabattoni) at TU Wien, Austria, about transformation of proofs in various non-classical logics.

6.2 National initiatives

We are involved, from 2022, in the ANR projet NARCO (Non-Aggregative Resource COmpositions - ANR-21-CE48-0011) with other research groups from LIG (Grenoble), VERIMAG (Grenoble) and LMF (Paris Saclay). The main goal of this project is the development of a methodology for modeling and verification of modular systems built from resources composed using non-aggregative operators.

During the last NARCO meeting, organized at Paris, D. Méry has given a talk on a recent work about "Bifurcation Logic: Separation Through Ordering" and B. Izart has presented the recent results about "Bi-Intuitionistic Bunched Implications logic: Beth Semantics and Axiomatization".

Let us notice that the blog "Focus Sciences du CNRS" presents an article about the ANR Oroject NARCOS entitled "L'art de modéliser des systèmes informatiques complexes" https://lejournal.cnrs.fr/nos-blogs/focus-sciences/lart-de-modeliser-des-systemes-informatiques-complexes.

From end of 2025 D. Larchey-Wendling is member of the ANR project CoqoPetri (2025–2028) with research groups from LaBRI (Bordeaux) and IRIF (Paris). It is in the continuation of the collaboration with the team of J. Leroux at LaBRI, about the mechanization in Rocq of the Mayr theorem and related ones concerning the decidability of the reachability relation in Petri nets.

We have also a collaboration with the Logic team at IRIT Toulouse and mainly with H. van Ditmarsch about the combination of knowledge and resources and in this context the development of new epistemic logics with separation connectives.

7 Dissemination

7.1 Promoting scientific activities

7.1.1 Scientific events: selection

Reviewer - reviewing activities
We regularly make reviews for major journals of our domain (Information and Computation, Journal of

Logic and Computation, Mathematical Structures in Computer Science, Theoretical Computer Science, etc...) and also for major conferences (AiML, CSL, IJCAR, etc...).

7.1.2 Leadership within the scientific community

D. Galmiche is (elected) member of the Steering Committee of the International Conference TABLEAUX since September 2023.

D. Galmiche is member of the Scientific Advisory Board of CHIST-ERA (programme for coordinated research on long-term ICT and ICT-based scientific challenges).

7.1.3 Scientific expertise

D. Galmiche was expert for the ANR (Committee CE 48).
D. Galmiche was President of a HCERES Evaluation Committee.

7.1.4 Research administration

D. Galmiche is member (elected) of the Scientific Council of Université de Lorraine (UL).
He is also (elected) member of the Board of this Council and of the expertise board for evaluation of UL's HDR.

7.2 Teaching - Supervision - Juries

7.2.1 Teaching

We teach Computer Science at Université de Lorraine in different departments. D. Galmiche gives lectures as Professor at FST, Faculty of Sciences, (Master - first year M1 and second year M2). D. Méry gives lectures as Assistant Professor at IUT Brabois (Bachelor, Master). As full-time researcher at CNRS D. Larchey-Wendling can give some lectures in Master.
We give some details about our main lectures in Master (M1 and M2) that can be related to our research activities.

- Master: D. Galmiche, *Logiques non-classiques et Preuves*, M2, FST, Nancy, France.
- Master: D. Galmiche, *Preuves de programmes*, M2, FST, Nancy, France.
- Master: D. Galmiche, *Sémantique, Preuves et Types*, M2, FST, Nancy, France.
- Master: D. Galmiche, *Logique et Modèles de Calcul*, M1, FST, Nancy, France.
- Master: D. Méry, *Preuves de programmes*, M2, FST, Nancy, France.
- Master: D. Larchey-Wendling, *Preuves et Déduction Automatisée*, M1, FST, Nancy, France.

7.2.2 Supervision

D. Galmiche and D. Méry supervise the PhD thesis of Benjamin Izart, since end of 2022, on the subject "Logiques de ressource non-aggregatives: modèles et calculs". PhD defence expected in 2026.

D. Galmiche and D. Méry supervise the PhD thesis of Brandon Hornbeck, since end of 2021, on the subject "Extensions of BI logic: semantics, proof calculi, proof transformations". PhD defence expected in 2026.

7.2.3 Juries

D. Galmiche was external reviewer for the PhD thesis of Han GAO, entitled "Combinaison des logiques modales et de la logique intuitionniste : calculs structurés et nouvelles logiques", supervised by Nicola Olivetti at University of Aix-Marseille (October 2025.

8 Scientific production

8.1 Major publications

[1] D. Galmiche, B. Hornbeck and D. Méry. 'Cut-free labelled calculi and decidability for intuitionistic sentential logic with identity'. In: *Journal of Logic and Computation* (2024). DOI: 10.1093/logcom/exae071. URL: https://hal.science/hal-04908823.

[2] H. van Ditmarsch, D. Galmiche and M. Gawek. 'An Epistemic Separation Logic with Action Models'. In: *Journal of Logic, Language and Information* 32 (2023), pp. 89–116. DOI: 10.1007/s10849-022-09372-z. URL: https://hal.science/hal-04237922.

[3] D. Galmiche and D. Méry. 'Labelled Tableaux for Linear Time Bunched Implication Logic'. In: *8th International Conference on Formal Structures for Computation and Deduction (FSCD 2023))*. Vol. 260. LIPIcs - Leibniz International Proceedings in Informatics 260. Rome, Italy, 2023. DOI: 10.4230/LIPIcs.FSCD.2023.27. URL: https://hal.science/hal-04404122.

[4] J.-R. Courtault, H. Van Ditmarsch and D. Galmiche. 'A public announcement separation logic'. In: *Mathematical Structures in Computer Science* 29.06 (2019), pp. 828–871. DOI: 10.1017/S0960129518000348.

[5] D. Galmiche, P. Kimmel and D. Pym. 'A substructural epistemic resource logic: theory and modelling applications'. In: *Journal of Logic and Computation* 29.8 (2019), pp. 1251–1287. DOI: 10.1093/logcom/exz024.

[6] D. Galmiche, M. Gawek and D. Méry. 'Beth Semantics and Labelled Deduction for Intuitionistic Sentential Calculus with Identity'. In: *6th International Conference on Formal Structures for Computation and Deduction, FSCD 2021*. Vol. 13. LIPIcs - Leibniz International Proceedings in Informatics. Buenos Aires, Argentina, July 2021, pp. 1–21. URL: https://hal.univ-lorraine.fr/hal-03563655.

[7] D. Galmiche and D. Méry. 'Labelled Cyclic Proofs for Separation Logic'. In: *Journal of Logic and Computation* 31.3 (2021), pp. 892–922. URL: https://hal.univ-lorraine.fr/hal-03563631.

[8] D. Larchey-Wendling. 'Constructive Decision via Redundancy-Free Proof-Search'. In: *Journal of Automated Reasoning* 64 (2020), pp. 1251–1287. DOI: 10.1007/s10817-020-09555-y.

[9] D. Larchey-Wendling. 'Synthetic Undecidability of MSELL via FRACTRAN Mechanised in Coq'. In: *6th International Conference on Formal Structures for Computation and Deduction (FSCD 2021)*. Buenos Aires, Argentina, July 2021. DOI: 10.4230/LIPIcs.FSCD.2021.18. URL: https://hal.inria.fr/hal-03280264.

[10] D. Larchey-Wendling and J.-F. Monin. 'The Braga Method: Extracting Certified Algorithms from Complex Recursive Schemes in Coq'. In: *Proof and Computation II*. WORLD SCIENTIFIC, Aug. 2021, pp. 305–386. DOI: 10.1142/9789811236488_0008. URL: https://hal.inria.fr/hal-03338785.

8.2 Publications of the year

International journals

[11] M. Bouvel, L. Cioni and B. Izart. 'The interval posets of permutations seen from the decomposition tree perspective'. In: *Order* 42 (8th Jan. 2025), pp. 459–479. DOI: 10.1007/s11083-024-09690-w. URL: https://hal.science/hal-03456772.

[12] T. Lyon, A. Ciabattoni, D. Galmiche, M. Girlando, D. Larchey-Wendling, D. Méry, N. Olivetti and R. Ramanayake. 'Internal and External Calculi: Ordering the Jungle without Being Lost in Translations'. In: *Bulletin of the Section of Logic of the Polish Academy of Sciences* 54.1 (6th June 2025). DOI: 10.18778/0138-0680.2025.02. URL: https://hal.science/hal-05526101.

International peer-reviewed conferences

[13] D. Galmiche, T. Lang, D. Méry and D. Pym. 'Bifurcation Logic: Separation Through Ordering'. In: *ELectronic Proceedings in Theoretical Computer Science (EPTCS)*. 20th Int. Conference on Theoretical Aspects of Rationality and Knwoledge, TARK 2025. Vol. 437. Dusseldorf, Germany, 27th Nov. 2025. DOI: 10.4204/EPTCS.437. URL: https://hal.science/hal-05526084.

8.3 Cited publications

[14] H. V. Ditmarsch, D. Galmiche and M. Gawek. 'A separation logic with histories of epistemic actions as resources'. In: *Logic, Language, Information, and Computation: 29th International Workshop, WoLLIC 2023*. Vol. 13923. Lecture Notes in Computer Science (LNCS). Halifax NS, Canada: Springer Nature Switzerland, 2023, pp. 161–177. DOI: 10.1007/978-3-031-39784-4_10. URL: https://hal.science/hal-04404180.

[15] D. Galmiche. 'Separation Logics: Semantics and Proofs - extended abstract'. In: 5th International Workshop on Automated Reasoning in Quantified Non-Classical Logics, ARQNL 2024. Vol. Vol. 3875. CEUR Workshop Proceedings, 2024, pp 1–4. URL: https://hal.science/hal-04909066.

[16] D. Galmiche, B. Hornbeck and D. Méry. 'Automated Proof Search in Intuitionistic Sentential Logic'. In: 5th International Workshop on Automated Reasoning in Quantified Non-Classical Logics, ARQNL 2024. Vol. vol. 3875. CEUR Workshop Proceedings, 2024, pp 24–37. URL: https://hal.science/hal-04909025.

[17] D. Galmiche, T. Lang and D. Pym. 'Minimalistic System Modelling: Behaviours, Interfaces, and Local Reasoning'. In: 16th EAI International Conference on Simulation Tools and Techniques, SIMUtools 2024. LNICST, Springer, 9th Dec. 2024. URL: https://hal.science/hal-04908946.

[18] D. Larchey-Wendling. 'Quasi Morphisms for Almost Full Relations'. In: *30th International Conference on Types for Proofs and Programs, TYPES 2024*. Copenhagen, Denmark, 30th May 2024. URL: https://hal.science/hal-04727817.

[19] D. Larchey-Wendling and J.-F. Monin. 'Proof Pearl: Faithful Computation and Extraction of μ-Recursive Algorithms in Coq'. In: *14th International Conference on Interactive Theorem Proving (ITP 2023). Leibniz International Proceedings in Informatics (LIPIcs), Schloss Dagstuhl - Leibniz-Zentrum für Informatik (2023*. Vol. 278. Białystok, Poland, 26th July 2023, 21:1–17. DOI: 10.4230/LIPIcs.ITP.2023.21. URL: https://hal.science/hal-04200527.

[20] J.-R. Courtault and D. Galmiche. 'A Modal Separation Logic for Resource Dynamics'. In: *Journal of Logic and Computation* 28.4 (2018), pp. 733–778. DOI: 10.1093/logcom/exv031.

[21] J. Courtault, D. Galmiche and D. Pym. 'A Logic of Separating Modalities'. In: *Theoretical Computer Science* 637.1 (2016), pp. 30–58. DOI: 10.1016/j.tcs.2016.04.040.

[22] Y. Forster and D. Larchey-Wendling. 'Certified Undecidability of Intuitionistic Linear Logic via Binary Stack Machines and Minsky Machines'. In: *CPP 2019 Proceedings of the 8th ACM SIGPLAN International Conference on Certified Programs and Proofs*. Cascais, Portugal: ACM Press, 2019, pp. 104–117. DOI: 10.1145/3293880.3294096.

[23] D. Galmiche, P. Kimmel and D. Pym. 'A Substructural Epistemic Resource Logic'. In: *7th Indian Conference on Logic and Its Applications, ICLA 2017*. Vol. 10119. Lecture Notes in Computer Science. Kanpur, Inde, 2017, pp. 77–91.

[24] D. Galmiche, M. Marti and D. Méry. 'Relating Labelled and Label-Free Bunched Calculi in BI Logic'. In: *28th Int. Conference on Automated Reasoning with Analytic Tableaux and Related Methods, Tableaux 2019*. Vol. 11714. Lecture Notes in Computer Science. Londres, Royaume-Uni: Springer, 2019, pp. 130–146.

[25] D. Galmiche and D. Méry. 'Labelled Connection-based Proof Search for Multiplicative Intuitionistic Linear Logic'. In: *International Workshop on Automated Reasoning in Quantified Non-Classical Logics, ARQNL 2018*. Vol. 2095. CEUR Workshop Proceedings. Oxford, Royaume-Uni, 2018, pp. 49–63.

[26] D. Larchey-Wendling. 'Constructive Decision via Redundancy-Free Proof-Search'. In: *Automated Reasoning. IJCAR 2018. Lecture Notes in Computer Science*. Vol. 10900. Automated Reasoning 9th International Joint Conference, IJCAR 2018, Held as Part of the Federated Logic Conference, FloC 2018, Oxford, UK, July 14-17, 2018, Proceedings. Oxford, Royaume-Uni, 2018, pp. 422–438. DOI: 10.1007/978-3-319-94205-6_28.

[27] D. Larchey-Wendling. 'Proof Pearl: Constructive Extraction of Cycle Finding Algorithms'. In: *Interactive Theorem Proving. ITP 2018. Lecture Notes in Computer Science*. Vol. 10895. ITP 2018: Interactive Theorem Proving. Oxford, Royaume-Uni, 2018, pp. 370–387. DOI: 10.1007/978-3-319-94821-8_22.

[28] D. Larchey-Wendling. 'Typing Total Recursive Functions in Coq'. In: *Interactive Theorem Proving. ITP 2017. Lecture Notes in Computer Science*. Vol. 10499. ITP 2017: Interactive Theorem Proving. Brasilia, Brésil, 2017, pp. 371–388. DOI: 10.1007/978-3-319-66107-0_24.

[29] D. Larchey-Wendling and Y. Forster. 'Hilbert's Tenth Problem in Coq'. In: *4th International Conference on Formal Structures for Computation and Deduction (FSCD 2019)*. Ed. by H. Geuvers. Vol. 131. Dortmund, Allemagne: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 2019, 27:1–27:20. DOI: 10.4230/LIPIcs.FSCD.2019.27.

[30] D. Larchey-Wendling and R. Matthes. 'Certification of Breadth-First Algorithms by Extraction'. In: *Mathematics of Program Construction. MPC 2019. Lecture Notes in Computer Science*. Vol. 11825. Porto, Portugal, 2019, pp. 45–75. DOI: 10.1007/978-3-030-33636-3_3.

[31] H. Van Ditmarsch, D. Galmiche and M. Gawek. 'An Epistemic Separation Logic with Action Models'. In: *9th Indian Conference on Logic and its Applications, ICLA 2021*. 9th Indian Conference on Logic and its Applications, ICLA 2021. Chennai, India, Mar. 2021. URL: https://hal.univ-lorraine.fr/hal-03563669.