MODERN COMPUTER ARITHMETIC
ANSWERS TO SELECTED EXERCISES

RICHARD P. BRENT AND PAUL ZIMMERMANN

Note: this document only contains answers to some exercises of the book. If an answer to some particular exercise you are interested in is missing, please contact the authors of the book at `MCA@rpbrent.com` or `Paul.Zimmermann@inria.fr`.

Exercise 1.1 Extend the Kronecker–Schönhage trick mentioned at the beginning of §1.3 to negative coefficients, assuming the coefficients are in the range $[-\rho, \rho]$.

Answer Each coefficient can take at most $2\rho + 1$ different consecutive values. Take a power $X = \beta^k > 2n\rho^2$ of the base β – where the degree of $A(x)$ and $B(x)$ is less than n – and multiply the integers $a = A(X)$ and $b = B(X)$, giving the integer c, which can now be negative. The coefficients from $C(x) = A(x)B(x)$ can be retrieved from the low degrees as follows. We have $c = c_0 \bmod X$, and we know $-n\rho^2 \le c_0 \le n\rho^2$. Let $d = c \bmod X$ with $0 \le d < X$; if $0 \le d \le n\rho^2$, we have $c_0 = d$, otherwise $c_0 = d - X$. Then we deduce c_1 similarly from $(c - c_0)/X$, etc.

Alternatively, we can perform the base-X decomposition of $c = C(X)$ from the high degrees, by rounding each coefficient to the nearest integer.

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