

4 Robustness issues: numerical issues, degenerate cases.

4.1 double arithmetic

4.1.1 Small questions

Let us use `double` arithmetic. For each of the following statements, answer whether it is true or false, and justify in less than one line.

$$a > b \Leftrightarrow a - b > 0 \quad (1)$$

$$(a * b) * c = a * (b * c) \quad (2)$$

$$a + b = b + a \quad (3)$$

$$a * (b + c) = a * b + a * c \quad (4)$$

$$x > y \Rightarrow \text{sqrt}(x) > \text{sqrt}(y) \quad (5)$$

$$(\text{for } x, y \geq 0) \quad x * x \geq y * y \Rightarrow x \geq y \quad (6)$$

$$a, b, c \text{ integers in } [-2^{20}, 2^{20}] \Rightarrow (a - b) * (a - c) = a * a + a * (c - b) - b * c \quad (7)$$

4.1.2 A function

What does the following function return when called on a `double` in the open interval $] -2^{50}, 2^{50}[$?

```
double WhoAmI{double x}
{
    double a = 3377699720527872.0;    // 2^50 + 2^51
    double s = x+a;
    double r = s-a;
    return r;
}
```

4.2 Circle intersection

Let C_1 and C_2 be two circles of respective centers (x_1, y_1) and (x_2, y_2) and respective radii r_1 and r_2 ,

4.2.1 Predicate

Write the predicate testing if C_1 and C_2 intersect as the sign of a polynomial in $x_1, y_1, r_1, x_2, y_2, r_2$.

4.2.2 Precision

Assume that the input data $x_1, y_1, r_1, x_2, y_2, r_2$ are integers in $[-2^b, 2^b]$, and that the computations are performed with `double`. For which values of b is the predicate guaranteed to give the correct result?

(Recall that according to IEEE754 norm, `double` are stored with 53 significant bits.)