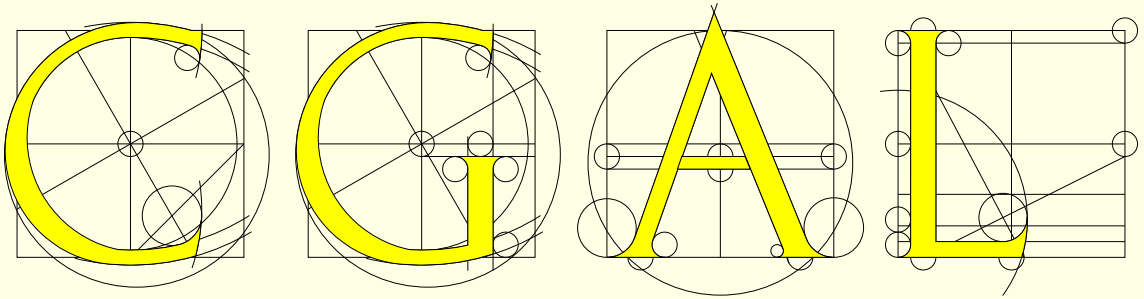
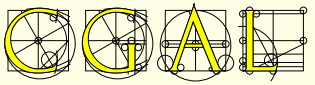


# *Computational Geometry Algorithms Library*

**www.cgal.org**

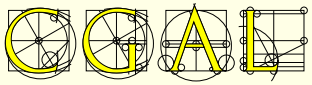
Monique Teillaud

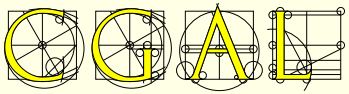


# Overview

- The CGAL Open Source Project
- Structure of CGAL
- The Kernel
- Numerical Robustness
- Contents of the Basic Library
- Flexibility
- Work in Progress



# The  Open Source Project

# Goals

- Promote the research in Computational Geometry (CG)
- *"make the large body of geometric algorithms developed in the field of CG available for industrial applications"*

$\Rightarrow$ **robust programs**

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## CG Impact Task Force Report, 1996

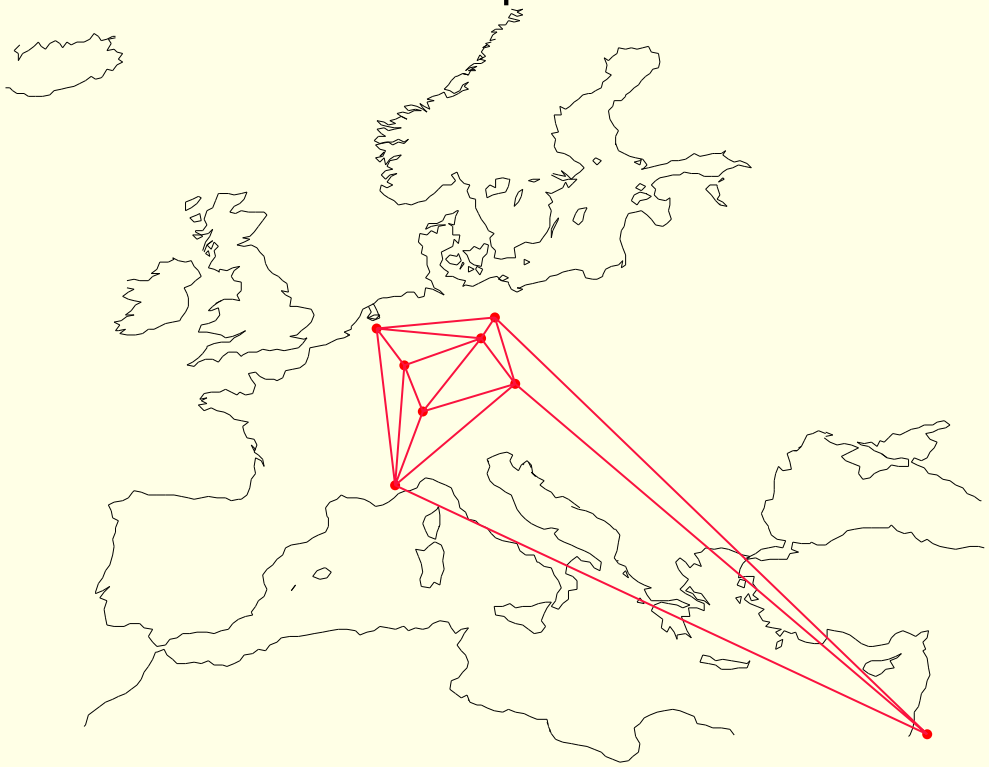
Among the key recommendations:

- Production and distribution of usable (and useful) geometric codes
- Reward structure for implementations in academia

# History

Development started 1995

Consortium of 8 European sites



RISC Linz
Utrecht University (XYZ Geobench)
INRIA Sophia Antipolis (C++GAL)
ETH Zürich (Plageo)
MPI Saarbrücken (LEDA)
Tel Aviv University
Freie Universität Berlin
Martin-Luther-Universität Halle

- Two ESPRIT LTR European Projects (1996-1999)
- Work continued after the end of European support in several sites.

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- January, 2003: **creation of Geometry Factory**

INRIAstartup
sells commercial licenses, support, customized developments

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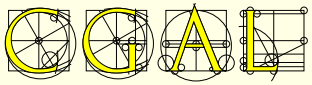
Release 3.0
**Open Source Project**





# License

- *kernel* under **LGPL**
- *basic library* under **QPL**
  - free use for OS code
  - commercial license needed otherwise

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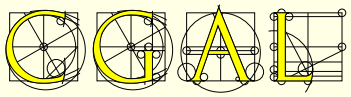
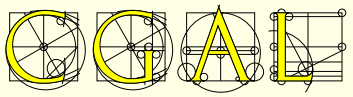
- A guarantee for CGAL users
- Allows CGAL to become a standard
- Opens CGAL for new **contributions**



# in numbers

- 350.000 lines of C++ code
- ∼2000 pages manual



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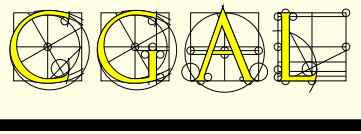
- CGAL 3.0.1: 6200 downloads (8 months)



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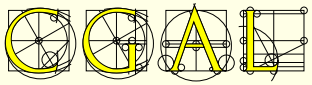
- 4000 subscribers to the announcement list (7000 for gcc)
- 800 users registered on discussion list (600 in gcc-help)
- 50 developers registered on developer list



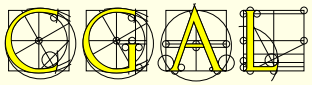


## Supported platforms

- Linux, Irix, Solaris, Windows
- Mac OS X (3.1)

- g++, SGI CC, SunProCC, VC7, Intel





# Development process

**Editorial Board** created in 2001.

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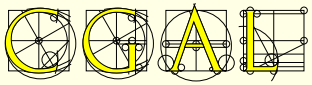
- decides about technical matters
- coordinates communication and promotion
- ...

Andreas Fabri (Geometry Factory)
Efi Fogel (Tel Aviv University)
Bernd Gärtner (ETH Zürich)
Michael Hoffmann (ETH Zürich)
Menelaos Karavelas (University of Notre Dame, **USA** → Greece)
Lutz Kettner (Max-Planck-Institut für Informatik)
Sylvain Pion (INRIA Sophia Antipolis)
Monique Teillaud (INRIA Sophia Antipolis)
Remco Veltkamp (Utrecht University)
Mariette Yvinec (INRIA Sophia Antipolis)





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- Own manual tools: LaTeX $\longrightarrow$ ps, pdf, html
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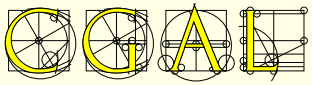
- Developer manual
- mailing list for developers
- 1-2 developers meetings per year, 1 week long

- 3 internal releases per week
  (1 per day before release)
- Automatic **test suites** running on all supported compilers/platforms





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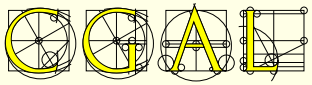

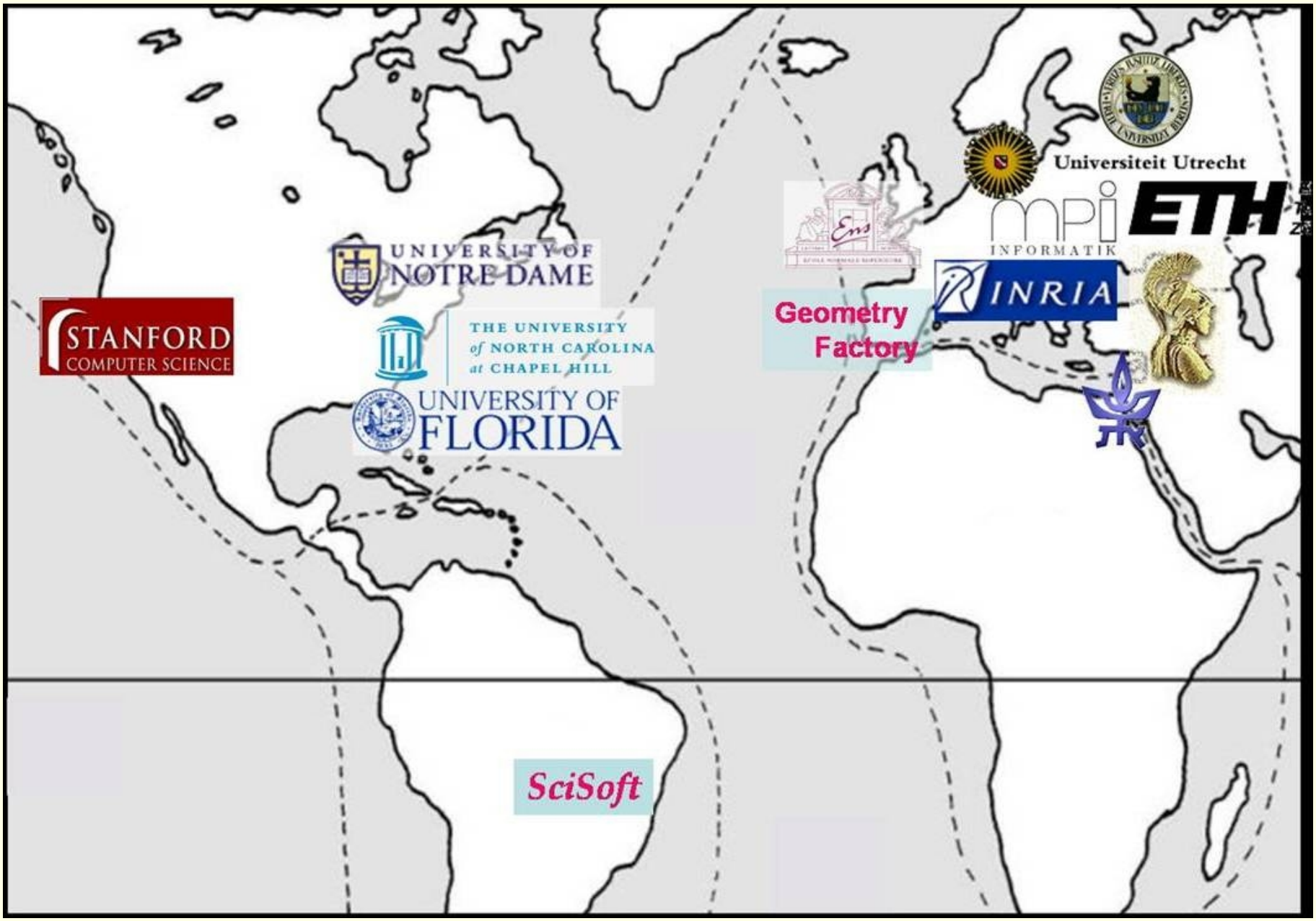
Universiteit Utrecht
ETH
INFORMATIK
INRIA
Geometry
Factory
UNIVERSITY OF
NOTRE DAME
STANFORD
COMPUTER SCIENCE
THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL
UNIVERSITY OF
FLORIDA
SciSoft

# Users

## Projects using CGAL

Leonidas J. Guibas' and co-workers, Stanford University.

Tamal K. Dey's and co-workers, The Ohio State University.

Nina Amenta and co-workers, The University of Texas at Austin.

Xiangmin Jiao, University of Illinois at Urbana-Champaign.
(Surface Mesh Overlay)

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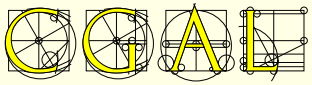
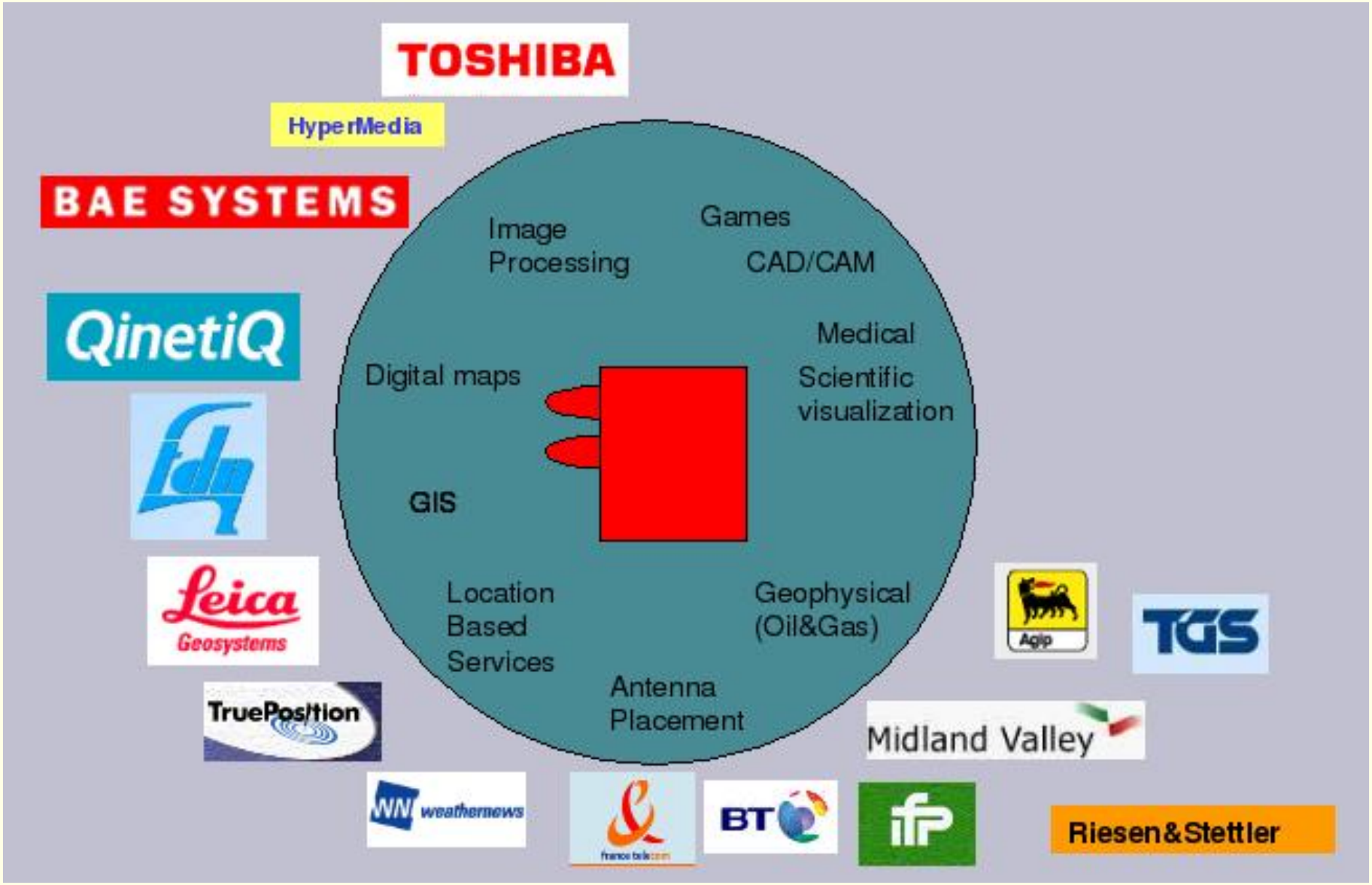
## Teaching

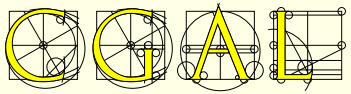
- Leo Guibas, Siu Wing Cheng, . . .





# Commercial customers of Geometry Factory



# Structure of 

**Basic Library**

Algorithms and Data Structures

**Kernel**

Geometric objects
Geometric operations

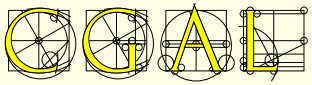
**core library**

configurations, assertions, ...

**Support Library**

Visualization
File I/O
NumberTypes
Generators
...





# The GOAL Kernel

# In the kernel

Elementary geometric objects

Elementary computations on them

**Primitives**
**2D, 3D, dD**
- Point
- Vector
- Triangle
- Iso_rectangle
- Circle

. . .

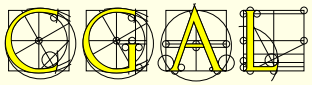
**Predicates**
- comparison
- Orientation
- InSphere

. . .

**Constructions**
- intersection
- squared distance

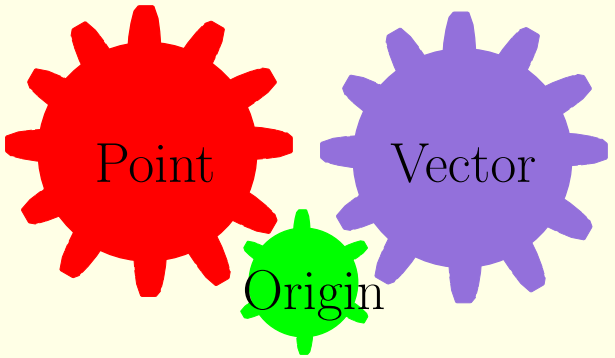
. . .





# Affine geometry

Point - Origin $\longrightarrow$ Vector
Point - Point $\longrightarrow$ Vector
Point + Vector $\longrightarrow$ Point



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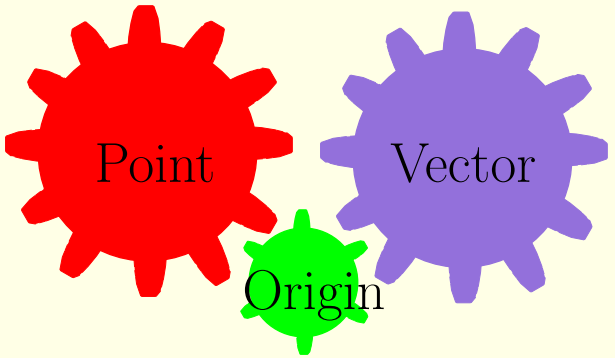


Point + Point **illegal**

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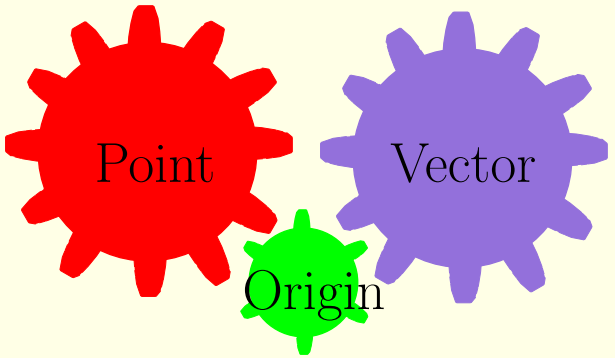


Point + Point **illegal**

midpoint(a,b) = a + 1/2 x (b-a)

# Kernels and Number Types

**Cartesian representation**

$$\text{Point} \left| \begin{array}{l} x = \frac{hx}{hw} \\ y = \frac{hy}{hw} \end{array} \right.$$

**Homogeneous representation**

$$\text{Point} \left| \begin{array}{l} hx \\ hy \\ hw \end{array} \right.$$

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Intersection of two lines

$$\begin{cases} a_1 x + b_1 y + c_1 = 0 \\ a_2 x + b_2 y + c_2 = 0 \end{cases}$$

$$(x, y) = \left( \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, -\frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \right)$$

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## Field operations

## Ring operations

# C++ Templates

CGAL::Cartesian< FT >
CGAL::Homogeneous< RT >

(CGAL::Simple_Cartesian)
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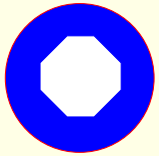
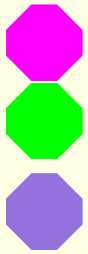


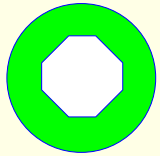
Cartesian Kernels : Field type



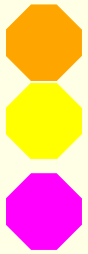
`double`

`Quotient<Gmpz>`

`leda_real`



Homogeneous Kernels : Ring type



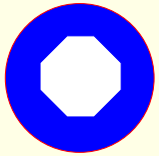
`int`

`Gmpz`

`double`

# C++ Templates

`CGAL::Cartesian< FT >` (`CGAL::Simple_Cartesian`)
`CGAL::Homogeneous< RT >` (`CGAL::Simple_Homogeneous`)



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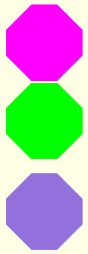

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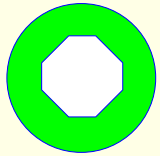

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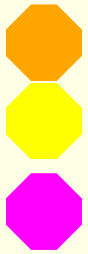


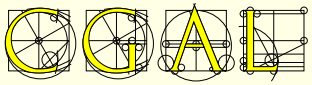
`int`

`Gmpz`

`double`

$\longrightarrow$ **Flexibility**

```
typedef double                          NumberType;
typedef Cartesian< NumberType >         Kernel;
typedef Kernel::Point_2                 Point;
```



# Numerical Issues

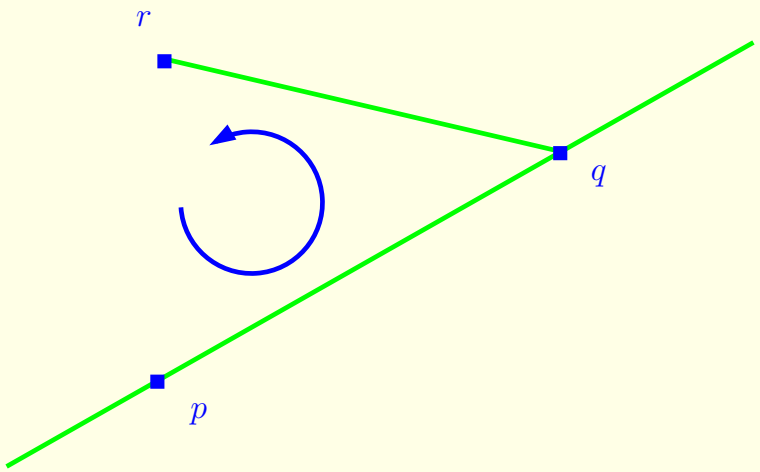
```
typedef CGAL::Cartesian<NT> Kernel;
NT sqrt2 = sqrt( NT(2) );

Kernel::Point_2 p(0,0), q(sqrt2,sqrt2);
Kernel::Circle_2 C(p,2);

assert( C.has_on_boundary(q) );
```

**OK if NT gives exact `sqrt`**
**assertion violation otherwise**

# Orientation of 2D points




$$
\begin{aligned}
orientation(p,q,r) &= \textbf{\textit{sign}}\left(\det\begin{bmatrix} p_x & p_y & 1 \\ q_x & q_y & 1 \\ r_x & r_y & 1 \end{bmatrix}\right) \\
&= \textbf{\textit{sign}}((q_x - p_x)(r_y - p_y) - (q_y - p_y)(r_x - p_x))
\end{aligned}
$$

$$p = (0.5 + x.u,\ 0.5 + y.u)$$
$$0 \leq\ x, y\ < 256,\ \ u = 2^{-53}$$
$$q = (12, 12)$$
$$r = (24, 24)$$

$p = (0.5 + x.u,\ 0.5 + y.u)$
$0 \leq\ x, y\ < 256,\ \ u = 2^{-53}$
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$orientation(p, q, r)$

evaluated with `double`

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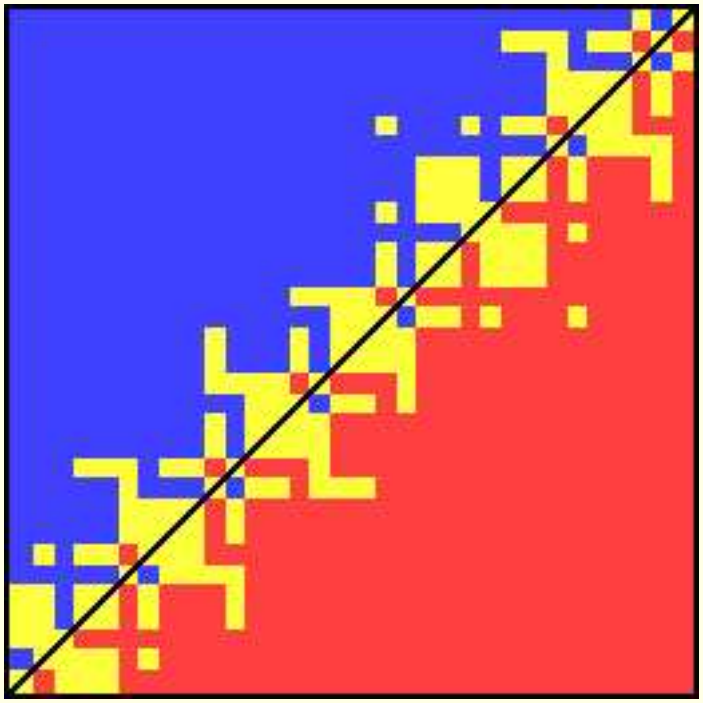
256 x 256 pixel image

$> 0$, $= 0$, $< 0$



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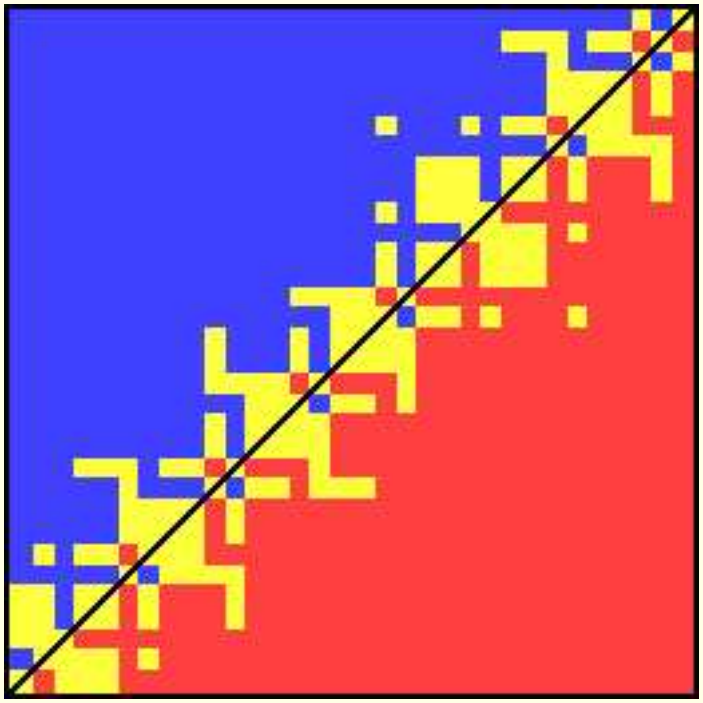
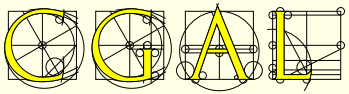


$\longrightarrow$ **inconsistencies** in predicate evaluations

`[Kettner, Mehlhorn, Pion, Schirra, Yap, ESA'04]`

# Numerical Robustness in 

imprecise numerical evaluations

$\longrightarrow$ non-robustness

combinatorial result

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**Exact Geometric Computation**

imprecise numerical evaluations

combinatorial result

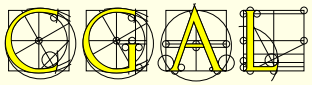
$\longrightarrow$ non-robustness

**Exact Geometric Computation**

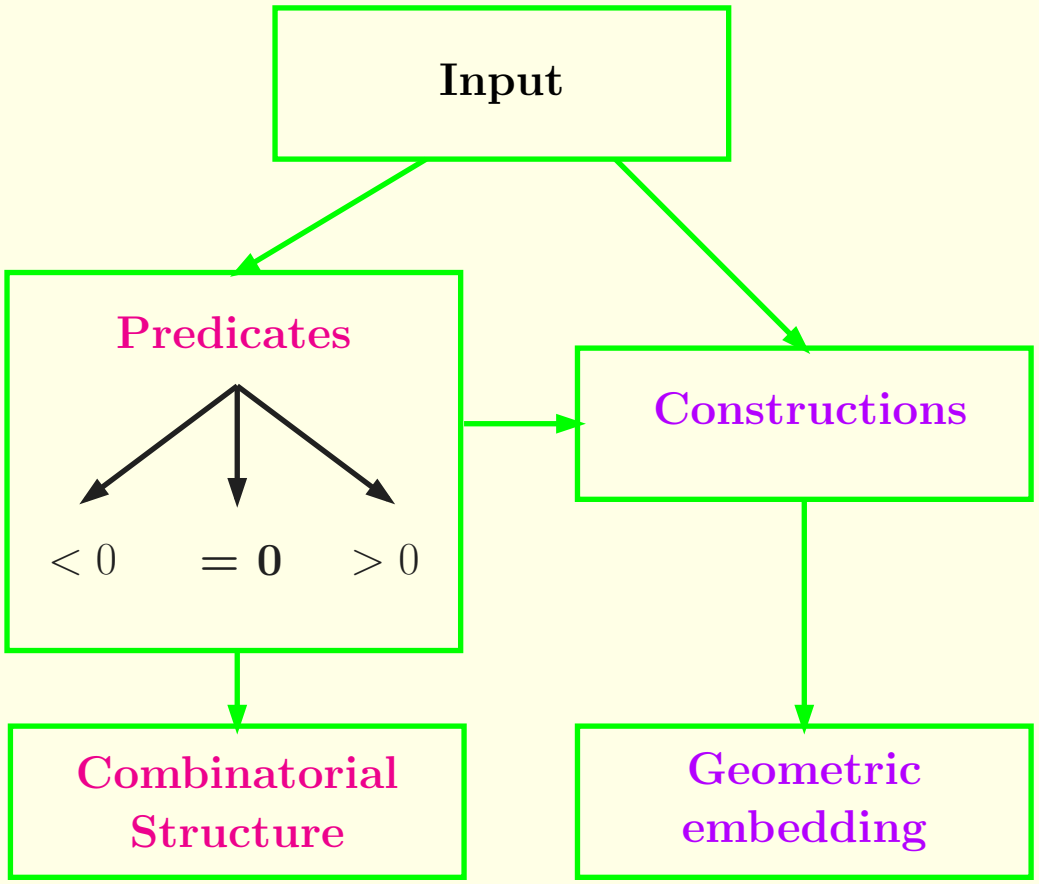
$\neq$

**exact arithmetics**

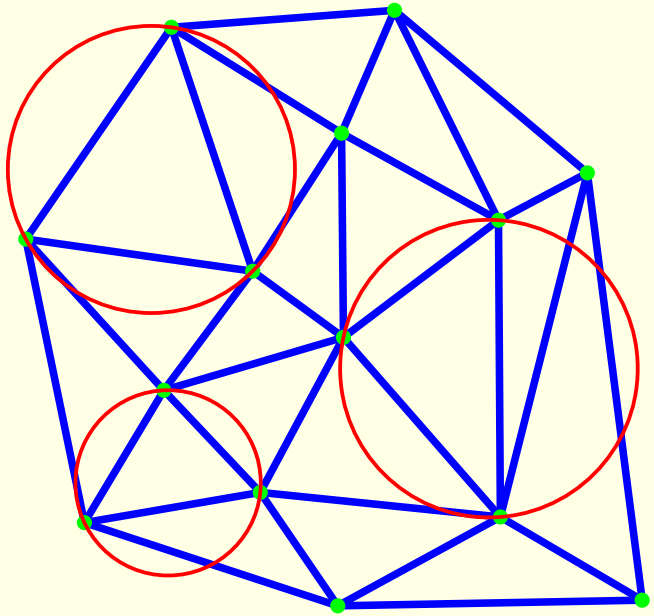




# Predicates and Constructions



## Delaunay triangulation



only **predicates** are used
*orientation*, *in_sphere*

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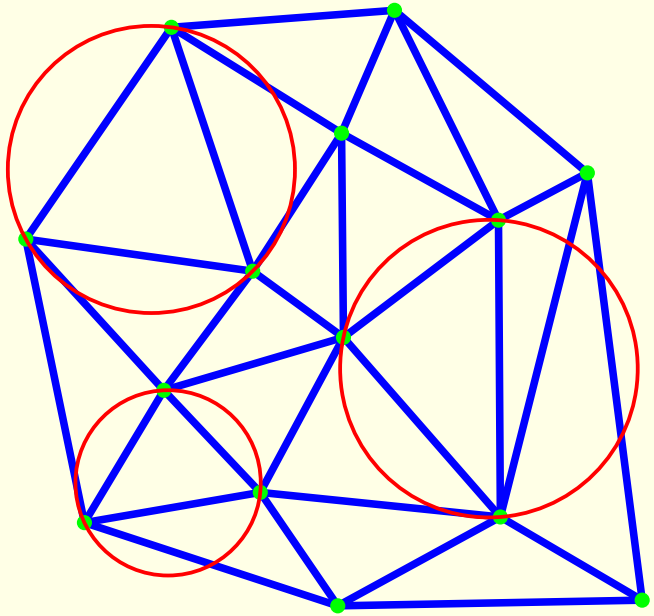

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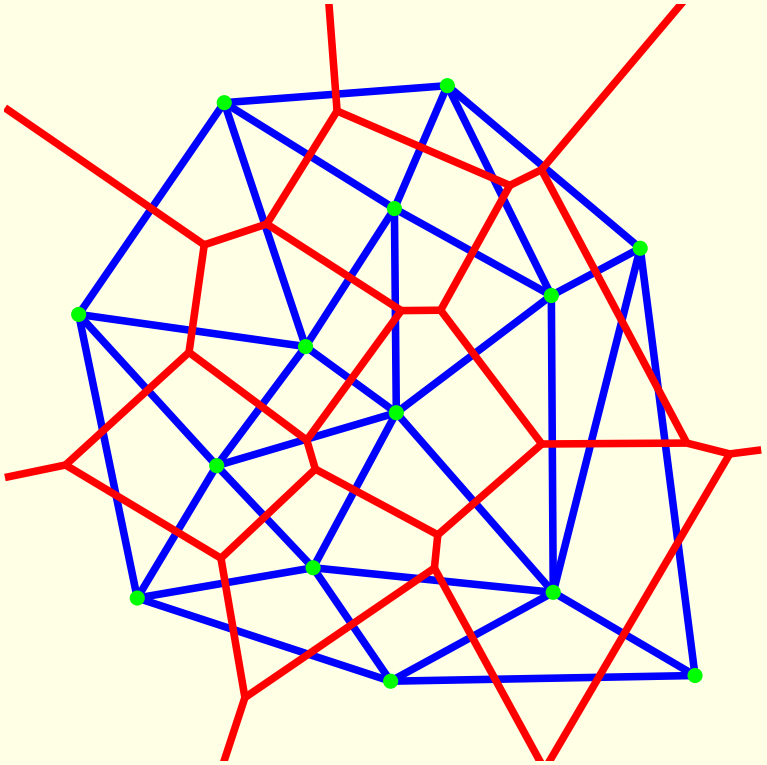
## Voronoi diagram



**constructions** are needed
*circumcenter*

# Arithmetic tools

- **Multiprecision integers**

Exact evaluation of signs / values of polynomial expressions with integer coefficients

**CGAL::MP_Float, GMP::mpz_t, LEDA::integer, ...**

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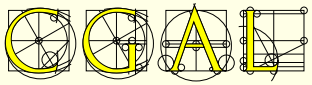
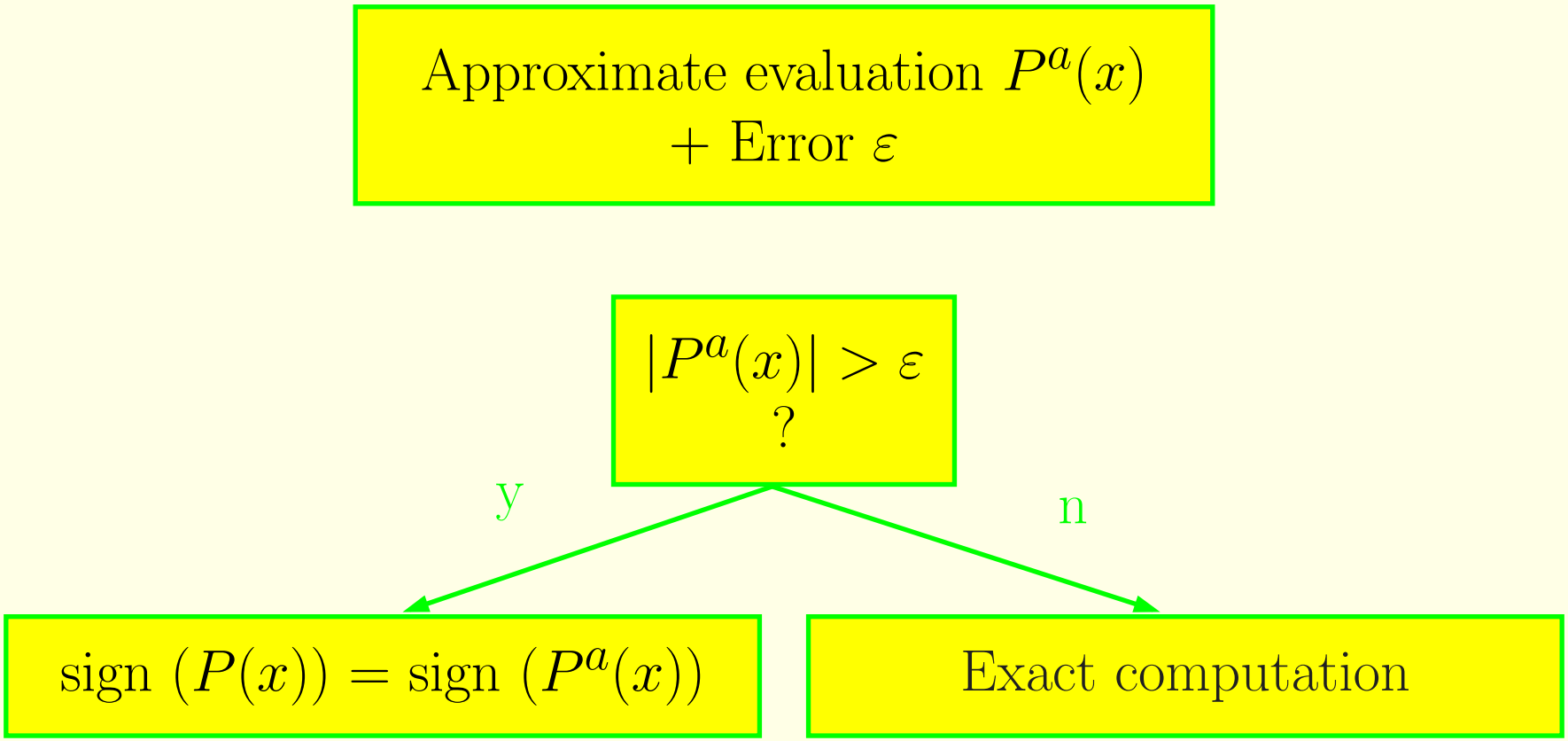
• **Algebraic numbers**
Exact comparison of roots of polynomials
**LEDA::real, Core::Expr** (work in progress in CGAL)



# Filtering Predicates

**sign** $(P(x))$ **?**



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Error bound precomputed faster

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Interval arithmetic more precise

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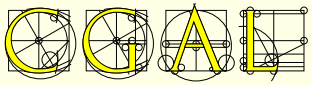
Dynamic only in CGAL 3.0 (Static filters exist but undocumented)
Static + Dynamic in CGAL 3.1

$\longrightarrow$ more generic generator also available for user's predicates





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Delays exact evaluation with **Exact_NT**:

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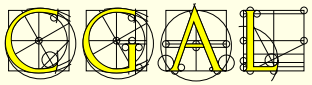
**CGAL::Lazy_kernel** in progress





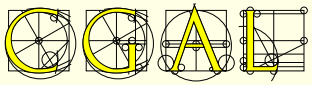
# Predefined kernels

**Exact_predicates_exact_constructions_kernel**
Filtered_kernel< Cartesian< Lazy_exact_nt< Quotient< MP_Float >>>>

**Exact_predicates_exact_constructions_kernel_with_sqrt**
Filtered_kernel< Cartesian< Core::Expr >>

**Exact_predicates_inexact_constructions_kernel**
Filtered_kernel< Cartesian< double >>





# Efficiency

**3D Delaunay triangulation**
CGAL-3.1-I-124

Pentium-M 1.7 GHz, 1GB
g++ 3.3.2, -O2 -DNDEBUG

1.000.000 random points

| | |
|---|---|
| Simple_Cartesian< double > | **48.1 sec** |
| Simple_Cartesian< MP_Float > | **2980.2 sec** |
| Filtered_kernel (dynamic filtering) | 232.1 sec |
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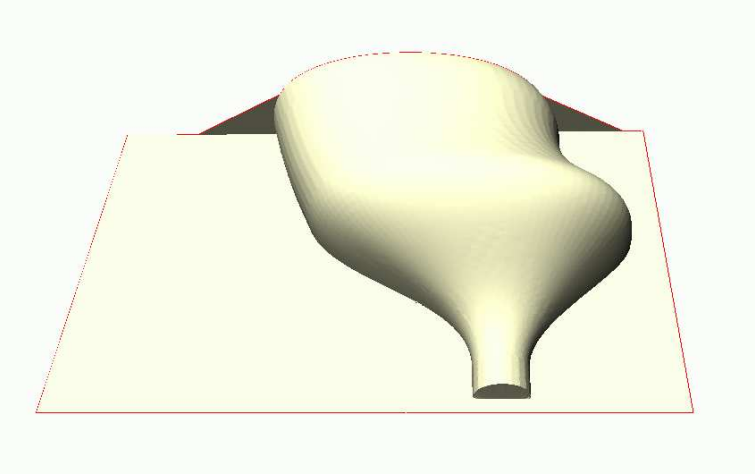

49.787 points (Dassault Systèmes)

| | |
|---|---|
| double | **loop !** |
| exact and filtered | $< 8$ sec |

## Robustness of Delaunay triangulations

Kernel and arithmetics $\longrightarrow$ Numerical robustness

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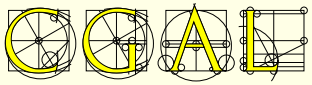
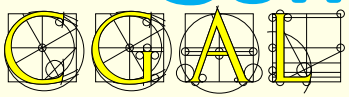
Kernel and arithmetics $\longrightarrow$ Numerical robustness

Algorithms $\longrightarrow$ explicit treatment of **degenerate cases**

Symbolic perturbation for 3D dynamic Delaunay triangulations
[Devillers Teillaud SODA'03]





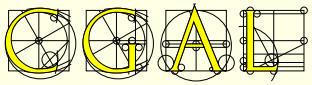
# Contents of the  Basic Library

# Convex Hull

[MPI]

- 5 different algorithms in 2D
- 3 different algorithms in 3D





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[INRIA]

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  (3.1 [Karavelas])
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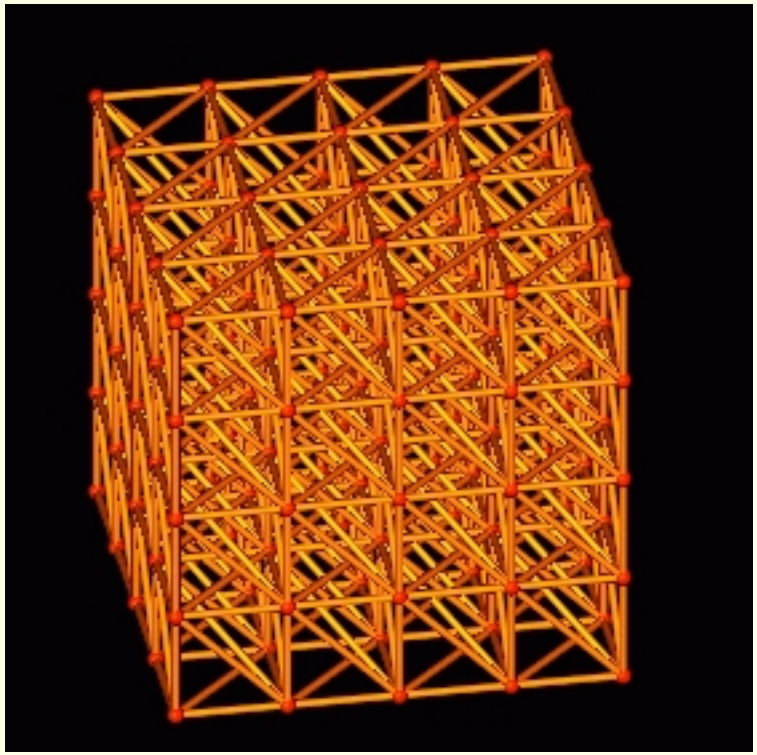

# Polyhedra

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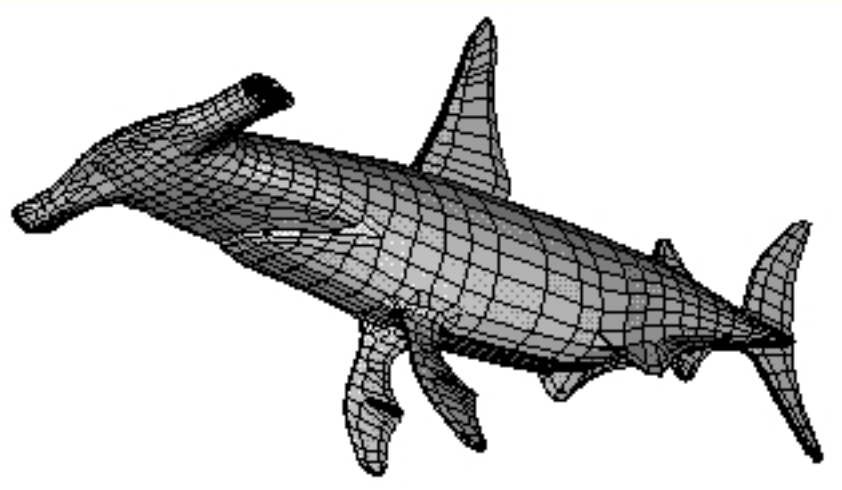
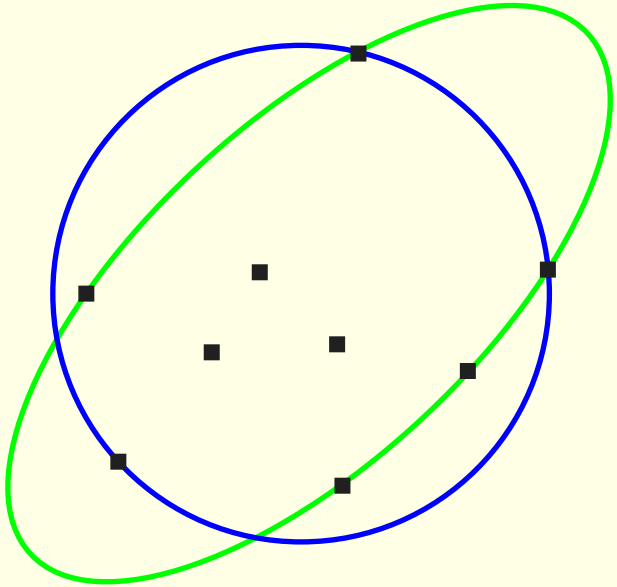
- 3D Nef polyhedra (3.1)



# Geometric Optimization

[ETH]

- Smallest enclosing circle and ellipse in 2D
- Smallest enclosing sphere in dD
- Largest empty rectangle
- . . .

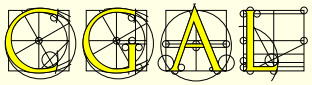


# Arrangements

[Tel-Aviv]

- Line segments or polylines
- Conic arcs with Leda or Core



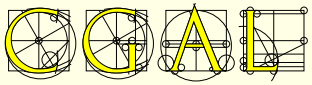


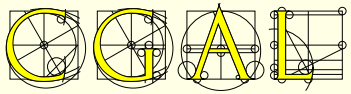
# Search Structures

Arbitrary dimension

- Range-tree, Segment-tree, kD-tree
- Window query
- Approximate nearest neighbors
- . . .





# Flexibility in the  Basic Library

# "Traits" classes

convex_hull_2<InputIterator, OutputIterator, **Traits**>
Polygon_2<**Traits**, Container>
Polyhedron_3<**Traits**, HDS>
Triangulation_3<**Traits**, TDS>
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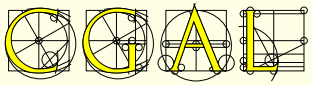
**Geometric traits classes** provide:
Geometric objects + predicates + constructors

- The **Kernel** can be used as a traits class for several algorithms
- Otherwise: **Default traits classes** provided
- The **user** can plug his own traits class





# Playing with traits classes

## Delaunay Triangulation

**Requirements** for a traits class:

- Point
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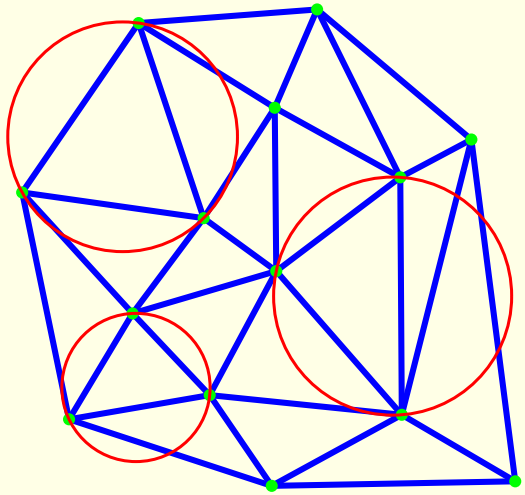

```
typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Delaunay_triangulation_2< K > Delaunay;
```

- 3D points: coordinates $(\mathbf{x}, \mathbf{y}, \mathbf{z})$
- orientation, in_circle: on $\mathbf{x}$ and $\mathbf{y}$ coordinates

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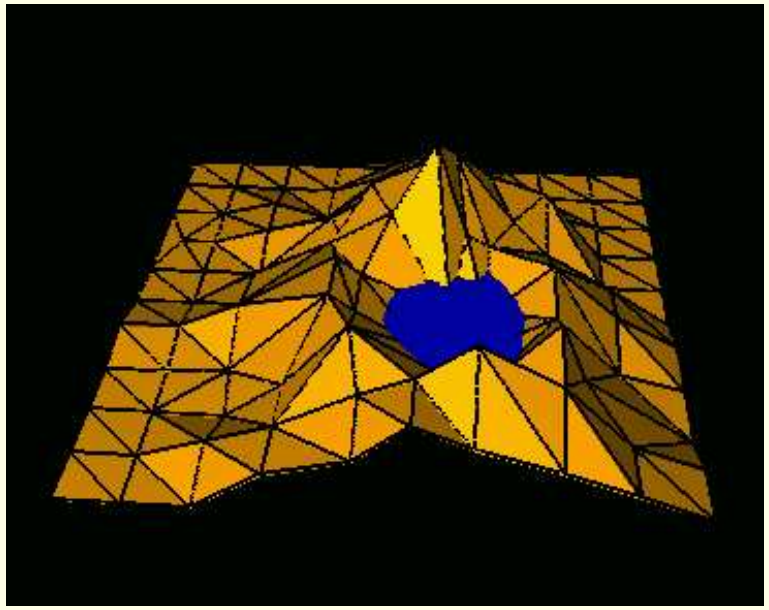
typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Triangulation_euclidean_traits_xy_3< K > Traits;
typedef CGAL::Delaunay_triangulation_2< Traits > Terrain;
```

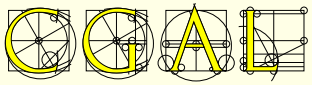


# More flexibility

The user can add information in vertices and cells

. . .





# Work in Progress

# Kinetic Data Structures

[Russel Karavelas]

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[Kettner Zomorodian]

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## Curved Kernel
Extension of the CGAL kernel
Algebraic issues
[Emiris Kakargias Pion Tsigaridas Teillaud SoCG'04]

. . .