Analysis and modeling of spatio-temporal activity in the brain

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01/11/06
1 Macroscopic scalp activity
   • Signal properties
   • Modeling of scalp activity

2 Mesoscopic neural populations
   • Brain structure
   • Visual hallucinations
   • General nonlocal interaction-model
   • Bifurcation analysis and linear response theory

3 Outlook
Outline

1. Macroscopic scalp activity
   - Signal properties
   - Modeling of scalp activity

2. Mesoscopic neural populations
   - Brain structure
   - Visual hallucinations
   - General nonlocal interaction-model
   - Bifurcation analysis and linear response theory

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Macroscopic scalp activity
Mesoscopic neural populations
Outlook

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Analysis and modeling of spatio-temporal activity in the brain
Cooperations related to this talk

- F. Kruggel (UC Irvine)
- C. Uhl (FH Ansbach)
- R. Friedrich (U Muenster)
- F. Atay (MPI Mathematics in the Sciences Leipzig)
- A. Longtin (U Ottawa)
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3. Outlook
Origin of scalp activity may be viewed as an electrical dipole in the brain:

→ electroencephalogram (EEG) or magnetoencephalogram (MEG)
Experimental result from a neuropsychological experiment

- 34 electric potentials measured on the scalp
- Stimulus of Kanizsa-figures at $t = 0$ ms, average over 400 repetitions and 16 subjects

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First results on dynamical modeling

- combination of low-dimensional projection and fit of differential equation system
- cost function \( V = V_{pca} + V_{dynamics} \) to be minimized (Uhl et al., PRE 1993)

- \( V_{pca} \): optimal projection on principal components
- \( V_{dynamics} \): least square fit of ordinary differential equation system
- \( \rightarrow \) slow time scale during cognitive processing, fast time scale in between
- problem: weight of projection and dynamics fit in method is arbitrary

(Uhl et al, Human Brain Mapping 1998)
Advanced modeling method

Minimizing the cost function

\[ V = V_{\text{pca}} + \varepsilon V_{\text{dynamics}} + \text{constraints} \ , \ \varepsilon \ll 1 \]

PCA: \[ v_i v_j = \delta_{ij} \text{ with spatial modes } v_k \]
Extended PCA: \[ w_i^\dagger w_j = \delta_{ij} \text{ with spatial modes } w_k, w_k^\dagger \]

\[ w_i = v_i + \varepsilon \sum_j c_{ij} v_j \ , \quad w_i^\dagger = v_i + \varepsilon \sum_j d_{ij} v_j \]

perturbation theory yields analytical expression for optimal spatial modes with

\[ V_{\text{pca}} = \sum_{i=1}^{M} \frac{\left\langle (q(t) - x_i w_i)^2 \right\rangle}{\left\langle q^2 \right\rangle} \]
\[ V_{\text{dynamics}} = \sum_{i=1}^{M} \frac{\left\langle (\dot{q} - \dot{x}_i w_i)^2 \right\rangle}{\left\langle \dot{q}^2 \right\rangle} \]

with the dynamical system \( \dot{x}_i = f[x_j] \)

(Hutt et al., PRE 1999)
Numerical example

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Signal properties
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Application to auditory evoked potentials

Experiment:

- subject hears a click tone of length 100 µs, average over $10^4$ trials
- EEG recorded with 32 channels from three subjects
Application of optimal projection

Fitted differential equation system:

\[ \dot{x}_1 = A_{11}x_1 + A_{12}x_2 + A_{13}x_1^2 + A_{14}x_2^2 + A_{15}x_1^3 + A_{16}x_1^2x_2 + A_{17}x_1x_2^2 + A_{18}x_2^3 \]

\[ \dot{x}_2 = A_{21}x_1 + A_{22}x_2 + A_{23}x_1^2 + A_{24}x_2^2 + A_{25}x_1^3 + A_{26}x_1^2x_2 + A_{27}x_1x_2^2 + A_{28}x_2^3 \]
Application of optimal projection
Three different subjects

First summary

What do we learn from these results?

- Linear superposition of few spatial modes allows for modeling most of the spatiotemporal dynamics of cognitive components, worse low-dimensional fit between components.
- Few modes are sufficient to model dynamics of brain → order parameters.
- Cognitive components reflect self-organized low-dimensional state, transitions are less organized and high-dimensional → chaotic itinerancy.
- Dynamic topology identical in all three subjects → common intrinsic functional structure.

Neuronal origin of this structure?
→ Investigation of spatiotemporal neural processes necessary.
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3. Outlook
here it is...

- Macroscopic scalp activity
- Mesoscopic neural populations
- Outlook

Brain structure
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.. and its substructure
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Explanation of visual hallucination

Experimental observations of visual hallucinations:

\[ z = c_1 re^{i\theta} \rightarrow w = c_2 \ln z = c_2 \cdot \ln c_1 r + ic_2 \theta \]

(from G.B. Ermentrout and J.D. Cowan, Biol. Cyb. 34(1979))
The neural population model of Ermentrout and Cowan (1979)

\[
\tau_E \frac{\partial E(x, t)}{\partial t} = -E(x, t) + S_e \left[ \int_{-\infty}^{\infty} \beta_{ee}(x - y)E(y, t) - \beta_{ie}(x - y)I(y, t) \, dy \right]
\]

\[
\tau_I \frac{\partial I(x, t)}{\partial t} = -I(x, t) + S_i \left[ \int_{-\infty}^{\infty} \beta_{ei}(x - y)E(y, t) - \beta_{ii}(x - y)I(y, t) \, dy \right]
\]

- \( E(x, t), \ I(x, t) \): excitatory and inhibitory neural activity at space point \( x \) and at time \( t \)
- \( S_e[V], \ S_i[V] \): nonlinear functions
- \( \beta_{ee}, \ \beta_{ei}, \ \beta_{ie}, \ \beta_{ii} \): spatial connectivity functions
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A generic nonlocal interaction model

\[
\frac{\partial V(x, t)}{\partial t} = h[V(x, t)] + \int_{\Omega} K(x - y)f[V(y, t)] + L(x - y)g[V(y, t)] \, dy
\]

\[
\int_{-\infty}^{\infty} K(x - y)V(y) \, dy = \sum_{n=0}^{\infty} (-1)^n K_n \frac{\partial^n V(x)}{\partial x^n}
\]

\[
K_n = \frac{1}{n!} \int_{-\infty}^{\infty} K(x)x^n \, dx
\]

equivalence of integral-differential equations and partial differential equations
Three examples

- **reaction-diffusion equation:**

  \[
  f[V] = V, \quad g[V] = 0, \quad K_n \rightarrow 0 \text{ for } n > 2 \quad \text{and} \quad K(x) = K(-x)
  \]

  \[
  \frac{\partial V(x, t)}{\partial t} = h[V(x, t)] + D \frac{\partial^2}{\partial x^2} V(x, t), \quad D = K_2
  \]

- **Swift-Hohenberg equation:**

  \[
  f[V] = V, \quad g[V] = 0, \quad h[V] = aV - bV^3, \quad K_n \rightarrow 0 \text{ for } n > 4,
  \]

  \[
  K(x) = K(-x)
  \]

  \[
  \frac{\partial V(x, t)}{\partial t} = \varepsilon V(x, t) - V^3(x, t) - \left(1 + \frac{\partial^2}{\partial x^2}\right)^2 V(x, t)
  \]

  with \( \varepsilon = 4K_4a/K_2^2|f_1| - 4K_4K_0/K_2^2 + 1 \)
Kuramoto-Sivashinsky equation:

\[ f[V] = fV^2 < 0, \quad g[V] = -V, \quad h[V] = -fK_0 V^2 - hV, \quad h \geq 0, \]
\[ K_n \to 0 \text{ for } n > 1, \quad L_n \to 0 \text{ for } n > 4 \text{ and } K(x) \neq K(-x), \quad L(x) = L(-x) \]
\[
\frac{\partial V(x, t)}{\partial t} = -\eta V - \frac{\partial^2}{\partial x^2} V(x, t) - \frac{\partial^4}{\partial x^4} V(x, t) - V \frac{\partial V}{\partial x}
\]

with \( \eta = L_0 + h \)

(Hutt, accepted by PRE)

reaction-diffusion equation, Swift-Hohenberg equation and Kuramoto-Sivashinsky equation are special cases of generic nonlocal interaction model
Physiological relevance of spatial connectivities

- $K(x, y)dy$, $L(x, y)dy$ gives the probability that there is an axonal connection to excitatory ($K(x)$) or inhibitory ($L(x)$) synapses between neuronal ensembles at spatial locations $x$ and $y$.
- Thus physiological experiments are necessary to trace single axons.
- Examination of many neuronal connections for computation of probability density.
- Presented results:
  - Intracortical connections in prefrontal cortex in monkeys.
  - Intracortical connections in visual cortex in monkeys.
  - Corticocortical connections in mice.
Prefrontal cortex in monkeys

(taken from J.B.Levitt et al., J.Comp.Neur.338,360(1993))

- short-range lateral connections in layer 5
- periodic connectivity in layer 3
  \[ K_e(x, y) \sim K_h(x - y) + K_p(x, y) \]
- \( K_p(x, y) \): inhomogenous, anisotropic and periodic
Visual cortex in monkeys

(taken from P.C. Bressloff, Physica D 185, 131(2003))

- tangential section through layer 2/3 showing lateral projections
- periodically spaced connection patches
  \[ K_e(x, y) \sim K_h(x - y) + K_p(x, y) \]
- \( K_p(x, y) \): inhomogenous, anisotropic and periodic
Corticocortical connections in mice


- **histogram of long-range connections**
- $\rightarrow$ gamma-distributed connectivity kernels
  
  $K_e(x, y) \sim K_h(x - y) \sim |x - y|^p e^{\frac{|x-y|}{\sigma}}$

- $K_h(x, y)$: assumption of homogeneity and isotropy
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Generic model subject to external global noise

\[
\frac{\partial V(x,t)}{\partial t} = h[V(x, t)] + \int_{\Omega} K(x - y) S_K[V(y, t)] + L(x - y) S_L[V(y, t)] \, dy + \kappa \xi(t).
\]

with Gaussian white noise \( \xi(t) \).
Analytical nonlinear stochastic bifurcation analysis:
Linear response

- Linearization:
  \[
  \frac{\partial u(x,t)}{\partial t} = s \int_{-\infty}^{\infty} K(x - y)u(y, t - \frac{|x-y|}{v}) \, dy + E(x, t)
  \]

- Greens function:
  \[
  u(x, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(x - y, t - \tau)E(y, \tau) \, dy \, d\tau
  \]
  analytical computation \( g(k, t) = \int_{-\infty}^{\infty} G(x, t) \exp(-ikx) \, dx \)

- Ansatz for stimulus:
  \[
  E(x, t) = \sum_{n=1}^{\infty} (M_n \cos(k_n x) + N_n \sin(k_n x))(\Theta(t) - \Theta(t - \Delta T))
  \]
  \[
  \rightarrow \text{for } t \geq \Delta T:\n  \]
  \[
  u(x, t) = \sum_{n=1}^{\infty} (M_n \cos(k_n x) + N_n \sin(k_n x)) \, I(k_n, t)
  \]
temporal part:

(Hutt and Atay, Chaos, Solitons and Fractals 2007)
localized stimulation for a typical spatial kernels
global stimulation

(Hutt and Atay, Chaos, Solitons and Fractals 2007)
reproduction of dynamic topology obtained from EEG by neural model

reproduction of evoked activity observed in EEG during psychological experiments

study of networks of neural population models subject to delays and internal noise