

ABSTRACT CANONICAL INFERENCE SYSTEMS

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Joint work with

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From unification theory

From
unification theory

to
theory unification

When objects (like theorems, algorithms, . . .) appears
to be variants from one to another

one would like to understand

why

and what is their shared structure

Instance of that problem:

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What is a most general unifier of

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Bendix	Knuth	Peterson	Stickel	Anantharaman
Mzali	Hsiang	Bachmair	Dershowitz	Barnett
Basin	Hesketh	Bellegarde	Lescanne	Bonacina
Buchberger	Bündgen	Bertling	Ganzinger	Kirchner
Chen	Christian	Comon	Josephson	Jouannaud
Doggaz	Fribourg	Gnaedig	Göbel	Gramlich
Hermann	Huet	Jantke	Thomas	Jouannaud
Marche	Kaplan	Remy	Kapur	Musser
Narendran	Zhang	Kuchlin	Lange	Lysne
Martin	Lai	Metivier	Mitchell	Mong
Purdom	Muller	Socher-Ambrosius	Stillman	Padawitz
Plaisted	Richter	Rusinowitch	Sattler-Klein	Snyder
Winkler	Yelick	Garland	Avenhaus	Madlener
Steinbach	Hintermeier	Lynch	Moreau	Strogova
Scharff	Shand	Linton	Otto	Toyama
Baader	Dubois	Okada	Goguen	Paterson
Struth	Aiguier	Barahmi	Stuber	Waldmann . . .

Towards good representations of axiomatic presentations

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Examples

Groups: Prove that $i(i(x)) = x$

$$x + e = x$$

$$x + (y + z) = (x + y) + z$$

$$x + i(x) = e$$

Groups: Same question with

$$x + e \rightarrow x$$

$$e + x \rightarrow x$$

$$x + (y + z) \rightarrow (x + y) + z$$

$$x + i(x) \rightarrow e$$

$$i(x) + x \rightarrow e$$

$$i(e) \rightarrow e$$

$$(y + i(x)) + x \rightarrow y$$

$$(y + x) + i(x) \rightarrow y$$

$$i(i(x)) \rightarrow x$$

$$i(x + y) \rightarrow i(y) + i(x)$$

AC: Solve the matching equation $x + x + y <_{AC} a + b + a + b + b$

$$\begin{aligned}w_1 + w_2 &= w_2 + w_1 \\w_1 + (w_2 + w_3) &= (w_1 + w_2) + w_3\end{aligned}$$

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as local as possible

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that we call here a completion process

What is a completion process?

Bruno Buchberger, RTA 85: *In order to solve an algorithmic problem for the ideal $\text{Ideal}(F)$ generated by F , first transform F into a certain canonical form G (which is called “Gröbner-basis” in /Buchberger 76/) such that $\text{Ideal}(F) = \text{Ideal}(G)$ and then solve the problem in G .*

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The process that transforms a given presentation to one which is more appropriate to the intended purpose.

Typical completion processes

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Aims of the talk

Provide the foundations to go

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Towards good-proof search

Structure of the talk

The set of canonical axioms, leading to good-proofs, is successively viewed as

- Formulæ that can appear as assumptions in minimal proofs
- Non-redundant theorems
- Conclusions of trivial proofs
- Limit of a completion process

and we conclude

Ordered proof system

- \mathbb{P} Proofs;
- \mathbb{A} Formulas;
- $\Gamma : \mathbb{P} \rightarrow 2^{\mathbb{A}}$ Assumptions;
- $\Delta : \mathbb{P} \rightarrow \mathbb{A}$ Conclusion;
- $\geq : \mathbb{P}^2$ Well-founded proof ordering.

Extension to sets:

$$\Gamma P = \bigcup_{p \in P} \Gamma p \quad (1)$$

$$\Delta P = \bigcup_{p \in P} \{\Delta p\} \quad (2)$$

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Assuming $a > b > c$ we have $p_2 \geq p_1$

Theories

The set of all the proofs of the formula c , starting from a set of formulæ A :

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The *theory* of a set of assumptions A :

$$\Theta A = \Delta \Pi A \quad (5)$$

Elementary play with the definitions

$$\Gamma \emptyset = \Delta \emptyset = \emptyset \quad (6)$$

$$\Gamma \Pi A \subseteq A \quad (7)$$

$$P \subseteq Q \Rightarrow \Gamma P \subseteq \Gamma Q \quad (8)$$

$$P \subseteq Q \Rightarrow \Delta P \subseteq \Delta Q \quad (9)$$

$$A \subseteq B \Rightarrow \Pi A \subseteq \Pi B \quad (10)$$

$$A \subseteq B \Rightarrow \Theta A \subseteq \Theta B \quad (11)$$

$$\Pi \Gamma \Pi A = \Pi A$$

By (7,10), $\Pi \Gamma \Pi A \subseteq \Pi A$.

Suppose $p \in \Pi A$. Then $\Gamma p \in \Gamma \Pi A$ by (8) and $p \in \Pi \Gamma \Pi A$, by definition.

Equivalent axiom sets

$$A \equiv B \Leftrightarrow \Theta A = \Theta B$$

Only what is used in proofs is needed:

$$A \equiv \Gamma \Pi A$$

Minimal proofs

$$\begin{aligned}\mu P &= \{p \in P \mid \neg \exists q \in P. \Delta q = \Delta p \wedge q < p\} \\ \Pi A &= \mu \Pi A\end{aligned}$$

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Because \geq is well-founded, we can prove as much using minimal proofs as with ordinary ones:

$$(\Theta A =) \Delta \Pi A = \Delta \Pi A$$

$\Delta \Pi A \supseteq \Delta \Pi A$: by monotonicity of Δ and since minimal proofs are proofs, i.e. $\Pi A \supseteq \Pi A$.
 $\Delta \Pi A \subseteq \Delta \Pi A$: for all $c \in \Delta \Pi A$, there exists p such that $p \in \Pi(A \vdash c)$. Since \geq is well-founded, there exists p' minimal, smaller than p that proves the same thing: $p' \in \Pi(A \vdash c)$, therefore $c \in \Delta \Pi A$.

Reduced systems

A set A of formulæ is *reduced* if

$$A = A^b =_{def} \Gamma \amalg A$$

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A reduced system can prove as much as the initial one:

$$A^b \equiv A$$

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Postulate A. [Monotonicity]

$$\Pi(A \vdash c) \neq \emptyset \Rightarrow \Pi(A \cup B \vdash c) \neq \emptyset$$

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Postulate B. [Reflexivity]

$$\Pi(\{a\} \vdash a) \neq \emptyset$$

Postulate C. [Closure]

$$\Theta\Theta A \subseteq \Theta A$$

First consequences

The theory generated by a set of formulæ A contains A :

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A set of formulæ A and its full theory ΘA support exactly the same theorems:

$$\Theta \Theta A = \Theta A \quad (\text{or } \Theta A \equiv A)$$

Canonical systems

A set A of formulæ is *saturated* if it supports all possible minimal proofs:

$$A \supseteq [\Theta A]^b$$

The set of formulæ that appear as assumptions of minimal proofs is called the **Canonical Basis**:

$$A^\# = [\Theta A]^b$$

Theorem — The canonical basis is a basis:

$$A^\# \equiv A$$

Consequence —

$$\Pi A^\# = \Pi \Theta A$$

The first definition of canonical axioms

Formulæ that are assumptions of minimal proofs

$$A^\# = [\Theta A]^b (=_{def} \Gamma \Pi \Theta A)$$

Another view point: Non-redundant theorems

Better proofs

Q is *better* than P :

$$P \sqsupseteq Q \Leftrightarrow \forall p \in P. \exists q \in Q. \Delta q = \Delta p \wedge p \geq q \quad (12)$$

A set of axioms B is a *simpler basis* than A when both can prove the same things, but the proofs made from B are better:

$$A \succsim B \Leftrightarrow A \equiv B \wedge \Pi A \sqsupseteq \Pi B$$

Theorem — A canonical basis is simpler:

$$A \equiv B \Rightarrow B \succsim A^\#$$

Redondant formulæ

The *redundant* formulæ in A are:

$$\rho A = \{r \in \mathbb{A} \mid A \succsim A \setminus \{r\}\} \quad (13)$$

When there exists redundant formulæ, without them we can prove as much, but the proofs are strictly better:

$$A \succsim A \setminus \rho A$$

Theorem — Redundant formulæ are not needed:

$$A^b = A \setminus \rho A$$

Corollary — The canonical basis is free of redondant formulæ:

$$A^\# = A^\# \setminus \rho A^\#$$

The second view of canonical axioms

Canonical axioms are non-redundant formulæ:

$$A^\# = A^\# \setminus \rho A^\#$$

Another point of view: deduction

Deduction: Soundness

We consider a *Deduction Mechanism* to be a mapping from sets of formulæ to sets of formulæ denoted $A \rightsquigarrow B$

A deduction mechanism is **sound** when

$$A \rightsquigarrow A' \Rightarrow \Theta A \supseteq \Theta A'$$

We will always consider *sound* deduction mechanism

Deduction: Persistency

A deduction derivation

$$A_0 \rightsquigarrow A_1 \rightsquigarrow \dots \rightsquigarrow A_i \rightsquigarrow \dots$$

The *limit* A_∞ of a derivation $\{A_i\}_i$ is its **persistent** formulæ:

$$A_\infty = \limsup_{i \rightarrow \infty} A_i = \bigcup_j \bigcap_{i > j} A_i$$

Deduction: Completeness

A derivation $\{A_i\}_i$ is *complete* if every theorem of A_0 eventually admits a persistent normal-form (i.e. belonging to $\Pi \Theta A_0$) proof:

$$\Theta A_0 \subseteq \Delta (\Pi A_\infty \cap \Pi \Theta A_0)$$

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For a complete derivation $\{A_i\}_i$

$$\Theta A_0 \subseteq \Theta A_\infty$$

The need for continuity

Postulate D. [Continuity]

$$\limsup_{i \rightarrow \infty} \Pi A_i = \Pi \limsup_{i \rightarrow \infty} A_i (= \Pi A_\infty)$$

A sufficient condition for a proof system to be continuous is to be finitely-based:

$$|\Gamma p| < \infty \text{ for all } p \in \mathbb{P}$$

i.e. any proof uses only a finite number of assumptions

Proposition – For sound and complete derivation we have:

$$\Theta A_0 = \Theta A_\infty$$

$\Theta A_0 \subseteq \Theta A_\infty$ By completeness

$\Theta A_0 \supseteq \Theta A_\infty$ By continuity:

$$\Theta A_\infty = \Delta \Pi A_\infty = \Delta \bigcup_j \bigcap_{i>j} \Pi A_i$$

So, if $c \in \Theta A_\infty$, then $c \in \Delta \Pi A_i = \Theta A_i$ for some i . But $\Theta A_i \subseteq \Theta A_0$ is guaranteed by soundness.

Reducing derivations

A derivation is *reducing* if its persistent equations are all reduced:

$$A_\infty = A_\infty^b$$

In other words, the limit does not contain any redundancy: $\rho A_\infty = \emptyset$.

Proposition — A derivation is canonical (i.e. reducing and complete) iff

$$A_\infty = A_0^\#$$

Simplifying Deduction

A deduction mechanism \rightsquigarrow is *simplifying* if it proves as much and the proofs only get better:

$$A \rightsquigarrow A' \Rightarrow \Theta A = \Theta A'$$

$$A \rightsquigarrow A' \Rightarrow \Pi A \sqsupseteq \Pi A'$$

This is denoted

$$\rightsquigarrow \sqsupseteq \rightsquigarrow$$

Expansion and contraction

A deduction step $A \rightsquigarrow A \cup B$ is an *expansion* provided

$$B \subseteq \Theta A$$

A deduction step $A \cup B \rightsquigarrow A$ is a *contraction* provided

$$A \cup B \lesssim A$$

Proposition — Expansions and contractions are sound

Fairness

Progressiveness A deduction mechanism δ is *progressive* if it makes every non-minimal proof better:

$$\delta(A) \subseteq \Theta A \quad (14)$$

$$\Pi A \setminus \Pi \Theta A \sqsubset \Pi (A \cup \delta(A)) \quad (15)$$

Fairness A derivation $\{A_i\}_i$ is *fair* for a progressive mechanism δ if all persistently progressive formulæ are derived:

$$\delta(A_\infty) \subseteq \bigcup_i A_i$$

Proposition — For continuous proof systems, simplifying fair derivations are complete.

Subproofs

Every formula a admits a trivial proof, by reflexivity. denoted \hat{a} , extended to sets by \hat{A}

We now assume the existence of a well-founded *subproof* (partial) order on proofs: $p[q] \triangleright q$, extended to sets:

$$P \triangleright Q \iff \forall q \in Q. \exists p \in P. p \triangleright q$$

We assume the following postulates for subproofs:

Subproof postulates

Postulate E. [Triviality] *Assumptions are subproofs:*

$$P \trianglelefteq \widehat{\Gamma P}$$

Postulate F. [Subproof] *Subproofs use a subset of the assumptions:*

$$P \trianglelefteq Q \Rightarrow \Gamma P \supseteq \Gamma Q$$

Postulate G. [Replacement] *Decreasing a subproof, decreases the whole proof:*

$$p \triangleright q \succ q' \Rightarrow \exists p' \in \mathbb{P}. p \succ p' \triangleright q' \quad (16)$$

A third characterisation of the canonical basis

The canonical basis is the set of conclusions of all trivial minimal proofs:

$$A^\# = \Delta(\Pi \Theta A \cap \widehat{\Theta A})$$

Completion

A minimal proof $p \in \Pi A$ is *critical* if it is not in normal form, but all its subproofs are:

$$\begin{aligned} p &\in \Pi A \setminus \Pi \Theta A \\ p \triangleright q &\Rightarrow q \in \Pi \Theta A \end{aligned}$$

The critical formulæ:

$$\nabla A = \{ \Delta p \mid p \text{ critical for } A \}$$

Bulk completion is a sequence of steps:

$$A \rightsquigarrow [A \cup \nabla A]^b \tag{17}$$

Fourth characterisation of the canonical basis

Fair completion is canonical.

For fair completion:

$$A_0^\# = A_\infty$$

Related works

- *Marc Aiguier, Diane Bahrami and Catherine Dubois:*
Axioms for rewriting theory (Rule'2000)
- *Marc Aiguier, Diane Bahrami:*
Une approche générique de la réécriture (TSI'2002)

Conclusions

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 - ★ to completion* processes
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- Explore other variants of the framework