Cette adorable personne est la tante de mon chapeau cordonnier.

Recouvre-toi, elle a la main droite et elle est une personne forte.

Peinture avec le majeur de la main droite.

Moi, plus j'ai un coeur, plus j'ai un cœur.

Bat, bat, bat.
Yet Another Dynamic Logic

Philippe de Groote
LORIA & Inria-Lorraine
A Type-theoretic reconstruction of DRT
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Motivation:

- to formalize DRT within Church's simple theory of type (aka, Higher-Order Logic), which will allow DRT and Montague semantics to rest on the same logical foundations.
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- to express dynamics using “static” primitives (in particular, to avoid the “destructive assignment” problem, which necessitates a LISP-like gensym operator).
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Challenge:

- to express dynamics using “static” primitives (in particular, to avoid the “destructive assignment” problem, which necessitates a LISP-like gensym operator).

Proposed solution:

- to interpret a sentence according to both its left and right contexts;
- to abstract these two kinds of contexts over the meaning of the sentences.
Typing the left and the right contexts
Typing the left and the right contexts

Montague semantics is based on Church’s simple type theory, which provides a full hierarchy of functional types built upon two atomic types:

- \( \iota \), the type of individuals (a.k.a. entities).
- \( \sigma \), the type of propositions (a.k.a. truth values).
Typing the left and the right contexts

Montague semantics is based on Church’s simple type theory, which provides a full hierarchy of functional types built upon two atomic types:

- $\iota$, the type of individuals (a.k.a. entities).
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We add a third atomic type, $\gamma$, which stands for the type of the left contexts.
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What about the type of the right contexts?

\[
\begin{align*}
\text{left context} & \quad \gamma \\
\text{right context} & \quad \gamma \rightarrow o
\end{align*}
\]
Semantic interpretation of the sentences
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Let $s$ be the syntactic category of sentences. Remember that we intend to abstract our notions of left and right contexts over the meaning of the sentences.
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$$[s] = \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o$$
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$$[s] = \gamma \to (\gamma \to o) \to o$$

Composition of two sentence interpretations
Semantic interpretation of the sentences

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$$[s] = \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o$$

Composition of two sentence interpretations

$$[S_1. S_2] = \lambda e\phi. [S_1] e (\lambda e'. [S_2] e' \phi)$$
Semantic interpretation of the syntactic categories
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Montague's interpretation

\[
\begin{align*}
[s] & \quad = \quad o \\
[n] & \quad = \quad \nu \rightarrow o \\
[np] & \quad = \quad (\nu \rightarrow o) \rightarrow o
\end{align*}
\]
Semantic interpretation of the syntactic categories

Montague’s interpretation

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[s] & = o \\
[n] & = \iota \to o \\
[np] & = (\iota \to o) \to o
\end{align*}
\]

may be rephrased as follows:

\[
\begin{align*}
[s] & = o \\
[n] & = \iota \to [s] \\
[np] & = (\iota \to [s]) \to [s]
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[np] = (\iota \rightarrow [s]) \rightarrow [s]
\] (1) (2) (3)

Replacing (1) with:

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Semantic interpretation of the syntactic categories

Montague’s interpretation

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\begin{align*}
[s] &= o \\
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\end{align*}
\] (1)(2)(3)

Replacing (1) with:

\[
[s] = \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o
\]

we obtain:

\[
\begin{align*}
[n] &= \iota \rightarrow \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o \\
[np] &= (\iota \rightarrow \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o) \rightarrow \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o
\end{align*}
\]
This interpretation results in handcrafted lexical semantics such as the following:

\[
[\text{every}] = \lambda n \psi \phi. (\forall x. \neg (n \ x \ e \ (\lambda e. \neg (\psi \ x \ (x::e) \ (\lambda e. \top)))))) \land \phi \ e
\]
This interpretation results in handcrafted lexical semantics such as the following:

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\]

which might seem a little bit involved.

Questions:

- is there a systematic way of obtaining the new lexical semantics from Montague's?
- can we find any “modular” presentation of the approach?
- is there some dynamic logic hidden in the approach?
A Dynamic Logic
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Let $\Omega \triangleq \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o$. We intend to design a logic acting on propositions of type $\Omega$. 
A Dynamic Logic

Let $\Omega \triangleq \gamma \to (\gamma \to o) \to o$. We intend to design a logic acting on propositions of type $\Omega$.

We share with DRT the two following assumptions:

- discourse composition is mainly conjunctive (roughly speaking, a discourse consists in the conjunction of its sentences);
- the main form of quantification is existential (it introduces referential markers).
A Dynamic Logic

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We share with DRT the two following assumptions:

- discourse composition is mainly conjunctive (roughly speaking, a discourse consists in the conjunction of its sentences);
- the main form of quantification is existential (it introduces referential markers).

Consequently, our logic will be based on conjunction and existential quantification (defined as primitives). The other connectives will be obtained using negation (a third primitive) and de Morgan's laws.
Conjunction
Conjunction

Conjunction is nothing but sentence composition. We therefore define:

\[ A \cap B \triangleq \lambda e \phi. A e (\lambda e. B e \phi) \]
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Existential quantification
Conjunction

Conjunction is nothing but sentence composition. We therefore define:

\[ A \sqcap B \triangleq \lambda e.\phi.\ A\ e\ (\lambda e.\ B\ e\ \phi) \]

Existential quantification

Existential quantification is canonically defined:

\[ \Sigma x.\ P\ x \triangleq \lambda e.\phi.\ \exists x.\ P\ x\ e\ \phi \]
Conjunction

Conjunction is nothing but sentence composition. We therefore define:

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Negation
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Existential quantification

Existential quantification is canonically defined:

\[ \Sigma x. P x \triangleq \lambda e \phi. \exists x. P x e \phi \]

Negation

Negation cannot be canonically defined because we do not want the continuation of the discourse to fall into the scope of the negation:

\[ \sim A \triangleq \lambda e \phi. \neg (A e (\lambda e. \top)) \land \phi e \]
Implication and Universal Quantification
Implication and Universal Quantification

These are defined using de Morgan’s laws:

\[ A \sqsubseteq B \triangleq \sim (A \sqcap \sim B) \]
\[ \Pi x. \, P \, x \triangleq \sim \Sigma x. \sim (P \, x) \]
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Embedding of first-order logic into dynamic logic
Implication and Universal Quantification

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\[ \Pi x. P x \triangleq \sim \Sigma x. (P x) \]

Embedding of first-order logic into dynamic logic

\[ \overline{R_i^n t_1 \ldots t_n} = \lambda e \phi. \overline{R_i^n t_1 \ldots t_n} \land \phi e \]
\[ \overline{\neg A} = \overline{\sim A} \]
\[ \overline{A \land B} = \overline{A} \land \overline{B} \]
\[ \overline{\exists x. A} = \overline{\Sigma x. A} \]
Implication and Universal Quantification

These are defined using de Morgan’s laws:

\[ A \implies B \triangleq \sim (A \land \sim B) \]
\[ \Pi x. P x \triangleq \sim \Sigma x. \sim (P x) \]

Embedding of first-order logic into dynamic logic

\[ R^n_i t_1 \ldots t_n = \lambda e \phi. R^n_i t_1 \ldots t_n \land \phi e \]
\[ \neg A = \sim A \]
\[ A \land B = \overline{A} \land \overline{B} \]
\[ \exists x. A = \Sigma x. \overline{A} \]

This embedding is such that, for every term \( e \) of type \( \gamma \):

\[ A \equiv A e (\lambda e. \top) \]
What about dynamics?
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No dynamics up to this point! In order to get a dynamic interpretation, we must put contexts (i.e., terms of type $\gamma$) at work.
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No dynamics up to this point! In order to get a dynamic interpretation, we must put contexts (i.e., terms of type $\gamma$) at work.

We need operations to update and access the contexts. As a first approximation, consider the contexts to be sets of individuals and add the following primitives to the system:

\[
\text{sel} : \gamma \rightarrow \iota
\]

where expressions such as $t :: e$ may be interpreted as $\{t\} \cup e$, and where $\text{sel}$ is a choice operator.
What about dynamics?

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We need operations to update and access the contexts. As a first approximation, consider the contexts to be sets of individuals and add the following primitives to the system:

\begin{align*}
_\vdash_ : \iota & \rightarrow \gamma \rightarrow \gamma \\
\text{sel} & : \gamma \rightarrow \iota
\end{align*}

where expressions such as $t : e$ may be interpreted as $\{t\} \cup e$, and where $\text{sel}$ is a choice operator.

We then define two translations of the (first-order) relational symbols:

\[ \overline{R^n_i} \triangleq \lambda x_1 \ldots x_n e \phi. R^n_i x_1 \ldots x_n \land \phi e \]

\[ \overline{R^n_i} \triangleq \lambda x_1 \ldots x_n e \phi. R^n_i x_1 \ldots x_n \land \phi(x_1 : \cdots : x_n : e) \]
Donkey sentence revisited
Donkey sentence revisited

Montague-like semantic interpretation:

\begin{align*}
[farmer] &= \text{farmer} \\
[donkey] &= \text{donkey} \\
[owns] &= \lambda OS. S (\lambda x. O (\lambda y. \text{own} x y)) \\
[beats] &= \lambda OS. S (\lambda x. O (\lambda y. \text{beat} x y)) \\
[who] &= \lambda RQx. Q x \land R (\lambda P. P x) \\
[a] &= \lambda PQ. \exists x. P x \land Q x \\
[every] &= \lambda PQ. \forall x. P x \supset Q x \\
[it] &= ???
\end{align*}
Donkey sentence revisited

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[it] &= ???
\end{align*}
\]

Dynamic interpretation:

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\begin{align*}
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[\text{beats}] &= \lambda OS. S (\lambda x. O (\lambda y. \underline{\text{beat}} x y)) \\
[\text{who}] &= \lambda RQx. Q x \sqcap R (\lambda P. P x) \\
[a] &= \lambda PQ. \Sigma x. P x \sqcap Q x \\
[\text{every}] &= \lambda PQ. \Pi x. P x \sqsupset Q x \\
[it] &= \lambda Pe\phi. P (\text{sel } e) e \phi
\end{align*}
\]
With the dynamic interpretation we have that:

\[
\texttt{[beats] [it] ([every] ([who] ([owns] ([a] [donkey])) [farmer])))}
\]
With the dynamic interpretation we have that:

$$\llbracket \text{beats} \rrbracket \llbracket \text{it} \rrbracket (\llbracket \text{every} \rrbracket (\llbracket \text{who} \rrbracket (\llbracket \text{owns} \rrbracket (\llbracket \text{a} \rrbracket (\llbracket \text{donkey} \rrbracket)))) (\llbracket \text{farmer} \rrbracket))$$

$\beta$-reduces to the following term (modulo de Morgan’s laws):

$$\lambda e. \phi. (\forall x. \text{farmer } x \supset (\forall y. \text{donkey } y \supset (\text{own } x y \supset \text{beat } x (\text{sel}(x::y::e)))))) \land \phi e$$