Reasoning about contexts in Lambek Grammars

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Lambda in the syntax

de Groote 2001a:

An asymmetry in Lambek grammars:

syntax: everyone: S/(DP\S)

semantics: λP∀x.Px : ⟨⟨e, t⟩, t⟩

“This asymmetry can be broken by ... allowing λ-terms on the syntactic side.”

Muskens 2003:

Word order and constituency should be dealt with on a separate level. I will discuss a grammatical formalism (Lambda Grammars) that allows one to combine signs that are sequences of λ-terms with the help of linear combinators (essentially closed pure λ-terms in which each abstractor binds exactly one variable).

Muskens 2001: Oehrle, Cresswell, Curry...

More on ACGs later...
Plan

• \(\text{NL}_{CL}\): Non-associative Lambek grammar with combinators
  – \(\text{CL} = \text{Combinatory Logic, Context Logic, Continuation Logic}\)
• Soundness and completeness for \(\text{NL}_{CL}\)
  – Conservative wrt NL
• \(\text{NL}_\lambda\): NL with lambdas (for now, a notational variant of \(\text{NL}_{CL}\))
  – Lambdek Grammar
• Examples, comparisons, interpretations
  – Extends Barker and Shan 2006 to nested contexts (related to Morrill et al.’s Discontinuous Lambek Grammar)
    * Quantificational binding
    * Scope-taking adjectives: same
  – Undelimited continuations (Moortgat et al.)? No, delimited.
  – Approximating ACGs in a Lambek grammar.
\textbf{NL}_{CL}
**NL\textsubscript{CL}: a two-mode, non-associative Lambek grammar**

- A set of atomic formula symbols $\mathcal{A} = \{\text{DP}, \text{S}, \ldots\}$
- Two modes, default ($\backslash$, $/$) and continuation ($\backslash\backslash$, $\slash\slash$)
- A set of formulas $\mathcal{F} \supset \mathcal{A}$ such that for all $A, B \in \mathcal{F}$
  
  $\mathcal{F} ::= A \backslash B \mid B / A \mid A \backslash\backslash B \mid B \slash\slash A$

- A set of structures $\mathcal{S} \supset \mathcal{F}$ such that for all $X, Y \in \mathcal{S}$
  
  $\mathcal{S} ::= X \bullet Y \mid X \circ Y \mid I \mid B \mid C$

- The usual logical rules (next slide)
- Three structural postulates (next slide after).
- Note: fusion formulas have been omitted for simplicity. They can be conservatively added to the logic in the natural way (Re- stall theorem 11.52). I have set the symbols $\bullet$ and $\circ$ in red to emphasize that they are structural connectives (KJB).
### Logical rules (sequent presentation)

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
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| \[
\begin{array}{c}
\Gamma \vdash A \\
\Sigma \vdash C
\end{array}
\] \rightarrow
\[
\begin{array}{c}
\Sigma [(\Gamma \bullet A B)] \vdash C
\end{array}
\] | \L |
| \[
\begin{array}{c}
\Gamma \vdash A \\
\Sigma \vdash C
\end{array}
\] \rightarrow
\[
\begin{array}{c}
\Sigma [(B/A \bullet \Gamma)] \vdash C
\end{array}
\] | \L |

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\end{array}
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Perfectly ordinary.
Structural rules for $NL_{CL}$:

Restall:

[\text{I} \text{ (Restall's '0')}] is “a zero-place punctuation mark” (p. 30), where punctuation marks “stand to structures in the same way that connectives stand to formulae” (p. 19).

\[
\frac{p}{p \circ \text{I}}
\]

\[
\frac{p \bullet (q \circ r)}{q \circ ((B \bullet p) \bullet r)} \quad B
\]

\[
\frac{(p \circ q) \bullet r}{p \circ ((C \bullet q) \bullet r)} \quad C
\]

- I is a right identity with respect to $\circ$
- B governs mixed commutativity involving $\bullet$ and $\circ$
- C governs mixed associativity involving $\bullet$ and $\circ$
- Other interpretations later
Example derivation of $\textit{John saw everyone}$

\[
\begin{align*}
\text{DP} \quad \text{((DP} \backslash \text{S})/\text{DP} \cdot \text{DP}) & \vdash \text{S} \\
\text{John} \cdot (\text{saw} \cdot \text{DP}) & \vdash \text{S} \quad \text{LEX} \\
\text{John} \cdot (\text{saw} \cdot (\text{DP} \circ \text{I})) & \vdash \text{S} \quad \text{B} \\
\text{John} \cdot ((\text{DP} \circ ((\text{B} \cdot \text{saw}) \cdot \text{I}))) & \vdash \text{S} \\
\text{DP} \circ ((\text{B} \cdot \text{John}) \cdot ((\text{B} \cdot \text{saw}) \cdot \text{I})) & \vdash \text{S} \quad \text{B} \\
((\text{B} \cdot \text{John}) \cdot ((\text{B} \cdot \text{saw}) \cdot \text{I})) & \vdash \text{DP} \backslash \text{S} \\
\text{S} \triangleright (\text{DP} \backslash \text{S}) \circ ((\text{B} \cdot \text{John}) \cdot ((\text{B} \cdot \text{saw}) \cdot \text{I})) & \vdash \text{S} \quad \text{R} \\
\text{everyone} \circ ((\text{B} \cdot \text{John}) \cdot ((\text{B} \cdot \text{saw}) \cdot \text{I})) & \vdash \text{S} \quad \text{LEX} \\
\text{John} \cdot ((\text{everyone} \circ ((\text{B} \cdot \text{saw}) \cdot \text{I})) & \vdash \text{S} \quad \text{B} \\
\text{John} \cdot (\text{saw} \cdot (\text{everyone} \circ \text{I})) & \vdash \text{S} \\
\text{John} \cdot (\text{saw} \cdot \text{everyone}) & \vdash \text{S} \\
\text{everyone}(\lambda x.\text{saw} \cdot x \cdot \text{j})
\end{align*}
\]
• Completely ordinary Curry-Howard labeling.
• The two usual classes of derivations for *Someone saw everyone*.
• Long distance scope-taking (*Someone asked everyone to leave*).

More interesting derivations later.
Soundness and completeness for $\text{NL}_{CL}$
Frames for $\text{NL}_{CL}$

A frame $\mathcal{F}$ consists of

- A (flat) set of points $\mathcal{P}$
- 3-place accessibility relations $R_\bullet$ and $R_\circ$
- 1-place predicates $I$, $B$, and $C$

Models

A model $\mathcal{M}$ for $\text{NL}_{CL}$ is a frame along with an evaluation relation $\vDash$ that satisfies the following:
Evaluation

\[ x \not\models B/A \iff \forall y, z. (R \cdot xyz \land y \not\models A) \rightarrow z \not\models B \]
\[ y \not\models A \setminus B \iff \forall x, z. (R \cdot xyz \land x \not\models A) \rightarrow z \not\models B \]
\[ (z \models A \bullet B \iff \exists x, y. R \cdot xyz \land x \models A \land y \models B) \]

\[ x \models B \\ A \iff \forall y, z. (R \circ xyz \land y \models A) \rightarrow z \models B \]
\[ y \models A \setminus B \iff \forall x, z. (R \circ xyz \land x \models A) \rightarrow z \models B \]
\[ (z \models A \circ B \iff \exists x, y. R \circ xyz \land x \models A \land y \models B) \]

\[ x \models 1 \iff x \in I \]
\[ x \models B \iff x \in B \]
\[ x \models C \iff x \in C \]
\[ z \models p \bullet q \iff \exists x, y. R \cdot xyz \land x \models p \land y \models q \]
\[ z \models p \circ q \iff \exists x, y. R \circ xyz \land x \models p \land y \models q \]
Frame conditions

For structural postulate $P$, construct $F(P)$ as follows (Restall 249):

Propositional variables: $F(p) = (p = x)$

Structural connectives, either

Zero-place: $F(I) = Ix$
$F(B) = Bx$
$F(C) = Cx$

Two-place: $F(X \bullet Y) = \exists y \exists z. R \cdot yzx \land F(X)[x := y] \land F(Y)[x := z]$
$F(X \odot Y) = \exists y \exists z. R \circ yzx \land F(X)[x := y] \land F(Y)[x := z]$

Then for a structural rule $P = \frac{\Sigma[\Gamma] \vdash A}{\Sigma[\Gamma']} \vdash A$ in which $\Gamma$ and $\Gamma'$ contain $p_1, p_2, \ldots, p_n$ as propositional variables,

$$F(P) = \forall x, p_1, p_2, \ldots, p_n. F(\Gamma') \rightarrow F(\Gamma)$$
Example: right identity

\[
\forall x p. (\exists y \exists z R \circ y z x \land F(p)[x := y] \land F(1)[x := z]) \rightarrow (p = x)
\]

\[
\forall x p. (\exists y \exists z R \circ y z x \land (p = x)[x := y] \land F(1)[x := z]) \rightarrow (p = x)
\]

\[
\forall x p. (\exists y \exists z R \circ y z x \land (p = y) \land F(1)[x := z]) \rightarrow (p = x)
\]

\[
\forall x p. (\exists y \exists z R \circ y z x \land (p = y) \land I x[x := z]) \rightarrow (p = x)
\]

\[
\forall x p. (\exists z R \circ p z x \land I z) \rightarrow (p = x)
\]
Abbreviations (in the style of Restall)

Structural rules:

\[
\frac{\Sigma[\Gamma] \vdash A}{\Sigma[\Gamma'] \vdash A} \equiv \frac{\Gamma}{\Gamma'}
\]

Implicit universals:

\[
\forall x, y, z.Rxyz \equiv Rxyz
\]

Implicit existentials, one-place:

\[
Rx(T)z \equiv \exists y. Rxyz \land Ty
\]

Implicit existentials, three-place:

\[
R_1x(R_2uv)z \equiv \exists y. R_1xyz \land R_2uvy
\]
Structural postulate: \[ \frac{p}{p \circ I} \]

\[ \frac{p \bullet (q \circ r)}{q \circ ((B \bullet p) \bullet r)} \]

\[ \frac{(p \circ q) \bullet r}{p \circ ((C \bullet q) \bullet r)} \]

Frame condition:

\[ R \circ x(I)y \leftrightarrow x = y \]

\[ R \circ q(R \bullet (R \bullet (B)p)r)x \leftrightarrow R \bullet p(R \circ qr)x \]

\[ R \circ p(R \bullet (R \bullet (C)q)r)x \leftrightarrow R \bullet (R \circ pq)r x \]

**Soundness and Completeness** (Restall theorems 11.20, 11.37):

\[ X \vdash A \] is provable in every model \( \mathcal{M} = (\mathcal{F}, \models) \) that satisfies the frame conditions iff in every model, \( \forall x \in \mathcal{F}, x \models X \rightarrow x \models A. \)
**NL_{CL} is conservative over the NL fragment**

Worry: continuations might introduce unwanted commutativity.

**Conservativity:** Let $X \vdash A$ be a sequent built only from the ingredients allowed in NL: $\slash, \bullet, \backslash$. If $\text{NL}_{CL}$ is conservative over the NL fragment, then $X \vdash A$ is provable in $\text{NL}_{CL}$ iff it is provable in NL.

- Using $\backslash L$, $\slash L$, $\backslash R$ or $\slash R$ introduces a connective that never goes away, so the final sequent will not be relevant.
- That leaves only the structural rule. It introduces a $\circ$ which eventually has to be eliminated, so applications of the structural rule have to come in matched pairs.
- We can use rules like $\backslash L$ and $\slash L$ to target an abstracted element. But we could have targeted the same element when it was in-situ, so nothing new can be derived.
- The proof proceeds by extending a falsifying NL model to a falsifying $\text{NL}_{CL}$ model.

Open question: decidability?
\textbf{NL}_\lambda
First (rough) resemblance: Morrill 1994

Scope-taking implemented by the interaction of

- a non-associative mode (here, ●);
- an associative mode (here, via C);
- a wrap mode (here, B)
2\textsuperscript{d} resemblance: Reductions in the $\lambda\mu$-calculus (de Groote 2001b)

$\mu$-left: \[ N(\mu\alpha.M) \leadsto \mu\beta.M[(\alpha X) := \beta(NX)] \]

$\mu$-right: \[ (\mu\alpha.M)N \leadsto \mu\beta.M[(\alpha X) := \beta(XN)] \]

$\frac{N \bullet (M \circ X)}{M \circ ((B \bullet N) \bullet X)}$ \hspace{1cm} B

$\frac{(M \circ X) \bullet N}{M \circ ((C \bullet X) \bullet N)}$ \hspace{1cm} C

- B allows a scope-taking element to hop leftwards
- C allows a scope-taking element to hop (up) rightwards

Connection both with lambda and with continuations; though in the $\lambda\mu$-calculus, continuations are undelimited.
Third resemblance: Embedding $\lambda$-terms into Combinatory Logic

Shönfinkel’s mapping (Barendregt 1984:152):

\[ \langle x \rangle \equiv x \quad \quad \quad A(x, x) \equiv I \]
\[ \langle MN \rangle \equiv \langle M \rangle \langle N \rangle \quad \quad \quad A(x, M) \equiv KM \quad (x \text{ not free in } M) \]
\[ \langle \lambda x.M \rangle \equiv A(x, \langle M \rangle) \quad \quad \quad A(x, MN) \equiv s(A(x, M))(A(x, N)) \]

where $sxyz = xz(yz)$, $Kxy = x$, and $Ix = x$ as usual. For example,

\[ \langle \lambda x\lambda y.yx \rangle = S(K(SI))(S(KK)I) \]

David Turner adds clauses, more efficient for linear terms:

\[ A(x, MN) \equiv B M(A(x, N)) \quad (x \text{ not free in } M) \]
\[ A(x, MN) \equiv C(A(x, M))N \quad (x \text{ not free in } N) \]

where $Bxyz = x(yz)$ and $Cxyz = xzy$. Now

\[ \langle \lambda x\lambda y.yx \rangle = B(Cl)I \]
Linear combinators into NL$_{CL}$:

Now adapt the mapping for NL$_{CL}$ (still written $\langle \cdot \rangle$).
Since all abstracts are linear, no need to mention S or K.

\[
\langle x \rangle \equiv x \\
\langle p \cdot q \rangle \equiv \langle p \rangle \cdot \langle q \rangle \\
\langle \lambda x . p \rangle \equiv \mathbb{A}(x, \langle p \rangle)
\]

\[
\mathbb{A}(x, x) \equiv I \\
\mathbb{A}(x, p \cdot q) \equiv (B \cdot p) \cdot \mathbb{A}(x, q) \quad (x \text{ not free in } p) \\
\mathbb{A}(x, p \cdot q) \equiv (C \cdot \mathbb{A}(x, p)) \cdot q \quad (x \text{ not free in } q)
\]

Derived inference rule:

\[
\frac{\Sigma[\Gamma[p]] \vdash A}{\Sigma[p \circ \langle \lambda x . \Gamma[x] \rangle] \vdash A} \lambda
\]
Details about $\Gamma[p]$ structures

As usual with $\lambda$, we pay for conceptual simplicity with some definitional complexity.

\[
\Gamma[p] ::= \ p \mid \lambda y. \Gamma[p] \mid q \bullet \Gamma[p] \mid \Gamma[p] \bullet q
\]

This $\lambda$ “abstracts” only over structures built from $\bullet$ and $\lambda$.

Allowed:

\[
\frac{A}{A \circ \lambda x.x} \quad \frac{A \bullet B}{A \circ \lambda x.(x \bullet B)} \quad \frac{\lambda x.(x \bullet B)}{B \circ \lambda y \lambda x.(x \bullet y)}
\]

Disallowed:

\[
\frac{A \circ B}{A \circ \lambda x.(x \bullet B)} \quad \frac{\lambda x.(x \bullet B)}{B \circ \lambda x \lambda y.(x \bullet y)}
\]

Crucially linear: $x$ fresh (distinct from every other symbol in $\Gamma$).
Example derivation of *John saw everyone*

\[\vdash S \quad \text{LEX}\]

\[
\begin{array}{c}
\frac{
DP \bullet ((DP \backslash S)/DP \bullet DP) \vdash S}{
John \bullet (saw \bullet DP) \vdash S}
\end{array}
\]

\[\lambda \quad \text{R}\]

\[
\begin{array}{c}
\frac{
DP \circ \lambda x(John \bullet (saw \bullet x)) \vdash S}{
\lambda x(John \bullet (saw \bullet x)) \vdash DP \backslash S}
\end{array}
\]

\[\frac{S \backslash (DP \backslash S) \circ \lambda x(John \bullet (saw \bullet x)) \vdash S}{
John \bullet (saw \bullet S \backslash (DP \backslash S)) \vdash S}
\]

\[\text{everyone}(\lambda x.\text{saw } x)\]
Examples, comparisons, interpretations
Compared to Barker and Shan 2006

- Like B&S, gives delimited continuations in a TLG setting
- Like B&S, uses a right identity (Restall’s Push and Pop)
- Unlike B&S, $NL_{CL}$ generalizes to nested contexts
- Unlike B&S, no explicit control over evaluation order (yet!)

Compared to Kiselyov and Shan 2007

- Like K&S, gives delimited continuations in an intuitionistic substructural logic
- Unlike K&S, describes an ambiguous language
  - K&S provide explicit reset operator to delimit scope
  - Here, in-situ scope-takers are self-limiting, e.g., a scope-taker like `everyone: S/ \ (DP \ S)` takes scope over any containing S constituent (easily constrained by standard techniques for regulating access to resources, see Barker and Shan for details on scope islands).
Generalizing to nested contexts (contexts inside contexts)

\[
\begin{align*}
p \bullet q \vdash A \\
\frac{(p \circ 1) \bullet q \vdash A}{(p \circ (C \bullet 1) \bullet q) \vdash A} & \quad \text{C} \\
\frac{p \circ ((C \bullet 1) \bullet (q \bullet 1)) \vdash A}{p \circ (q \circ ((C \bullet (C \bullet 1)) \bullet 1)) \vdash A} & \quad \text{C} \\
\frac{p \bullet q \vdash A}{p \circ \lambda x(x \bullet q) \vdash A} & \quad \text{\lambda} \\
\frac{p \circ (q \circ \lambda y \lambda x(x \bullet y)) \vdash A}{p \circ (q \circ \lambda y \lambda x(x \bullet y)) \vdash A} & \quad \text{\lambda}
\end{align*}
\]

- \( \lambda x(x \bullet q) \) is a context: \( p \)'s delimited continuation relative to the proof of \( A \).
- \( \lambda y \lambda x(x \bullet y) \) is a context inside a context: the delimited continuation of \( q \) relative to the context \( \lambda x(x \bullet q) \).

\[ p \bullet q \vdash A \]

\[ (p \circ 1) \bullet q \vdash A \]

\[ p \circ ((C \bullet 1) \bullet q) \vdash A \]

\[ p \circ ((C \bullet 1) \bullet (q \bullet 1)) \vdash A \]

\[ p \circ (q \circ ((C \bullet (C \bullet 1)) \bullet 1)) \vdash A \]

\[ p \bullet q \vdash A \]

\[ p \circ \lambda x(x \bullet q) \vdash A \]

\[ p \circ (q \circ \lambda y \lambda x(x \bullet y)) \vdash A \]
Example: *Everyone said he left:*

\[
\begin{align*}
\text{he} & \quad \lambda R \lambda x. Rxx : (\text{DP} \downarrow \text{S}) \downarrow (\text{DP} \downarrow (\text{DP} \downarrow \text{S})) \\
\text{DP} \bullet (\text{said} \bullet (\text{DP} \bullet \text{left})) & \vdash \text{S} \\
\text{DP} \circ \lambda x (x \bullet (\text{said} \bullet (\text{DP} \bullet \text{left}))) & \vdash \text{S} \\
\lambda x (x \bullet (\text{said} \bullet (\text{DP} \bullet \text{left}))) & \vdash \text{DP} \downarrow \text{S} \\
\text{DP} \circ \lambda y \lambda x (x \bullet (\text{said} \bullet (y \bullet \text{left}))) & \vdash \text{DP} \downarrow (\text{DP} \downarrow \text{S}) \\
\lambda y \lambda x (x \bullet (\text{said} \bullet (y \bullet \text{left}))) & \vdash \text{DP} \downarrow (\text{DP} \downarrow \text{S}) \\
\text{DP} \downarrow (\text{DP} \downarrow (\text{DP} \downarrow \text{S})) & \vdash \text{DP} \downarrow \text{S} \\
\text{DP} \downarrow (\text{DP} \downarrow (\text{DP} \downarrow \text{S})) & \vdash \text{DP} \downarrow \text{S} \\
\lambda y \lambda x (x \bullet (\text{said} \bullet (y \bullet \text{left}))) & \vdash \text{DP} \downarrow \text{S} \\
\text{DP} \downarrow (\text{DP} \downarrow (\text{DP} \downarrow \text{S})) & \vdash \text{DP} \downarrow \text{S} \\
\lambda x (x \bullet (\text{said} \bullet (\text{he} \bullet \text{left}))) & \vdash \text{DP} \downarrow \text{S} \\
\lambda x (x \bullet (\text{said} \bullet (\text{he} \bullet \text{left}))) & \vdash \text{DP} \downarrow \text{S} \\
\text{S} \downarrow (\text{DP} \downarrow \text{S}) & \vdash \text{S} \\
\text{everyone} \circ \lambda x (x \bullet (\text{said} \bullet (\text{he} \bullet \text{left}))) & \vdash \text{S} \\
\text{everyone} \bullet (\text{said} \bullet (\text{he} \bullet \text{left})) & \vdash \text{S} \\
\text{everyone} ((\lambda R \lambda x. Rxx)(\lambda y \lambda x. \text{said(left x) y})) & = \text{eo}(\lambda z. \text{said(lft z) z}) \\
\text{(cf. Morrill, Fadda & Valentín 2007:52 : } \lambda R \lambda x. Rxx : ((\text{S} \uparrow \text{DP}) \uparrow \text{DP}) \downarrow (\text{S} \uparrow \text{DP}))
\end{align*}
\]
• Dowty 2007: duplication in lexicon, so no contraction in logic
• Unlike Dowty, pronoun not restricted to scoping over VP-oids
• Weak crossover currently unexplained (see Barker and Shan 2006)
Example: *Everyone read the same book*:

\[
\begin{align*}
\text{everyone} & \equiv \lambda f (\lambda y \cdot \text{read}(\text{the}(f(\text{book})))) \\
\end{align*}
\]

(Barker in press: parasitic scope: *same* scopes over *everyone*'s scope)
• as concatenation mode

\[ A \vdash C/B \quad (A \cdot B) \vdash C \quad B \vdash A\setminus C \]
as plug mode (delimited continuations)

\[ A \vdash C \parallel B \quad (A \circ B) \vdash C \quad B \vdash A \backslash C \]

- \( A \backslash C \): a \( C \) missing an \( A \) somewhere inside (Morrill: \( C \uparrow A \))
- \( A \circ B \): An \( A \) plugged into the hole in \( B \).
- \( C \parallel B \): a \( C \) missing a \( B \) at its top. (Morrill: \( B \downarrow C \))

\( A \backslash C \) is a delimited continuation.
Structural equivalence

\[ \Gamma[A] = A \circ \Gamma[\ ] \]
Delimited, not undelimited continuations

- Logically, undelimited continuations have type $A \rightarrow \bot$
- Computationally, functions that never return
- A strategy: Barker 2002, de Groote 2001b, Bernardi and Moortgat 2007: identify $\bot$ with some useful result type, such as $S$.
- Limitation: special steps needed to allow a scope-taking element to change the result type of the expression it takes scope over.
- Logically, delimited continuations have type $A \backslash B$
- Computationally, delimited continuations are composable
- Conjecture (Barker 2004): natural language makes use of only delimited continuations
  - In-situ $wh$: $Q \backslash (DP \backslash S)$: John saw who?
  - Focus particles (Barker 2004)
  - Pied-Piping (Moortgat circa 2000) $which$: $Rel \backslash (DP \backslash DP)$: the book [the author of which] Alice admires
ACGs
• Allow $\lambda$ to abstract over $\circ$ as well as over $\bullet$ by adding two postulates. This makes the $\bullet$ mode and the $\circ$ mode symmetric.

• Let $SAW$ abbreviate $\lambda y \lambda x (y \circ (\text{saw} \bullet x))$

• Let $EVERYONE$ abbreviate $\lambda \kappa (\text{everyone} \circ \kappa)$

• Remember, “$x \circ f$” is value $\circ$ context, argument $\circ$ functor.

\[
\frac{
\vdash S}{
\text{ACG-ABBREV}}
\]

So $\bullet$ corresponds to phenogrammar, $\circ$ corresponds to techtogrammar, and the abbreviations characterize the relationship between the levels.


de Groote, Philippe. 2001b. Type raising, continuations, and classical logic. Amsterdam Colloq


Morrill, Glyn, Mario Fadda, and Oriol Valentín. Incomplete draft only, 2 August 2007. Discontinuous Lambek Grammar. ESSLLI, Dublin.