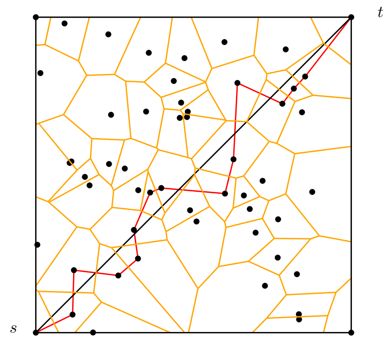
# 3 Probabilistic analyses: randomized algorithms, evenly distributed points.

## 3.1 Area of incident triangles

Let $X$ be Poisson point process of intensity $n$ in the plane. What is the expected area of the union of triangles incident to a point of $X$?

## 3.2 Voronoi Path

Let $X$ be a set of $n$ points inside the square $[0,1]^2$ and add to $X$ the four corners of the square. Let $s=(0,0)$ et $t=(1,1)$. Imagine a point moving on the line segment from $s$ to $t$, its nearest neighbor in $X$ is $s=v_0$ at the beginning, then it changes to another point $v_1 \in X$. We denote $s=v_0, v_1, v_2 \ldots v_{k-1}, v_k = t$ the sequence of nearest neighbors of the moving point. The polygonal line $v_0, v_1, v_2 \ldots v_{k-1}, v_k$ is called the Voronoi path.




### 3.2.1 Delaunay

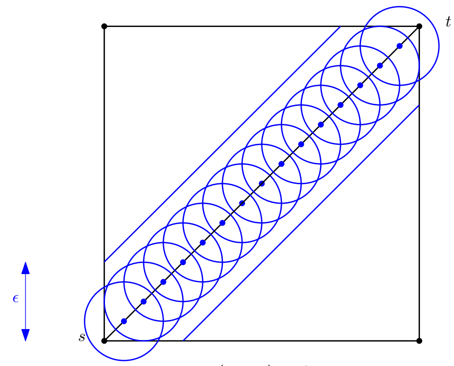
Is $v_i v_{i+1}$ a Delaunay edge? (Prove or give a counter-example).

### 3.2.2 Length

Give if possible upper and lower bound on the length (the sum of the lengths of all relevant edges) of the Voronoi path. Draw examples for these bounds.

### 3.2.3 Strip

In this question, assume that $X$ is a set of $n$ random points in the square (plus the four corners). The lines $y=x+\epsilon$ and $y=x-\epsilon$ describe a strip around the diagonal of the square and we want to study the probability that the Voronoi path leave this strip.
We cover the diagonal with disks of radius $\frac{\epsilon}{2}$ centered at points $\left(\frac{j\epsilon}{4}, \frac{j\epsilon}{4}\right)$ with $j$ integer in $\left[1, \frac{4}{\epsilon}-1\right]$.




- Prove that at a point $(x,x)$ on the diagonal the circle centered at $(x,x)$ of radius $\epsilon$ contains at least one of the circle of the diagonal covering.
- Deduce an upper bound of the required probability. (The classical inequality $1-t \leq e^{-t}$ could be useful).
- Find some (smallest possible) value of $\epsilon$, depending on $n$, such that this probability is exponentially small (as a function of $n$).

# Exam

Do not forget: your choice of project/ article presentation is due November 22nd.