On the topology of the cells of the 3D visibility complex

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Projet ISA

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Brief history

- The 2D visibility complex:
  - Introduced by Pocchiola and Vegter [PV96]
  - Optimal algorithms designed for polygonal and smooth convex obstacles [PV96]
  - Optimal algorithms for discs and polygons implemented by Angelier

- The 3D visibility complex :
  - Studied by Durand [DDP96,D99]
  - Full complex: theoretically huge size, intricate structure and topology
  - 1-dimensional skeleton: smaller and simpler. Algorithm designed and implemented for polyhedral scenes by Durand and Duguet
  - Expected size of complex: linear for realistic models
  - Better understanding of its topology and structure still needed

Our approach

- Balls of various radii.

$$
\begin{array}{c} \text{cells of the} \\ \text{visibility} \\ \text{complex} \end{array}
\longrightarrow
\begin{array}{c} \text{sets} \\ \text{of} \\ \text{lines} \end{array}
\longrightarrow
\begin{array}{c} \text{cells of the trace} \\ \text{of an arrangement} \\ \text{of quadrics} \\ \text{on a hypersurface} \end{array}
$$

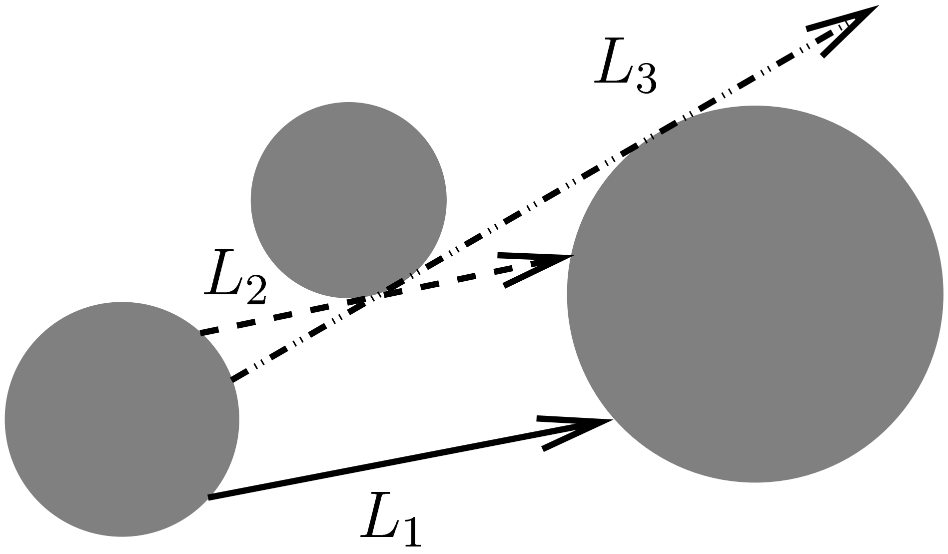
- Parameterization of the intersection of a quadric Q_B and the Plücker hypersurface Π.

First results

- Lines tangent to one ball: $\mathbb{S}^2 \times \mathbb{S}^1$
- Lines tangent to two balls: $\mathbb{S}^1 \times \mathbb{S}^1$ (torus)
- Extension of Pellegrini's ray-shooting structure [P93]

The 3D visibility complex : definition

- In the space of maximal free line segments
- Partition into connected components of segments touching the same objects



The Plücker coordinates of a line

- Mapping between lines in $\mathbb{R}^3$ and points in $\mathbb{P}^5(\mathbb{R})$

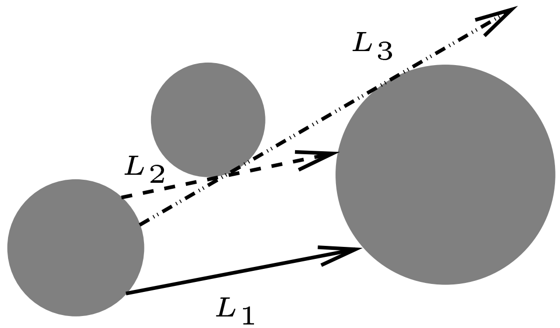
$$\begin{array}{c}\text{line with direction } \vec{s} \\ \text{passing through point } P\end{array} \quad \mapsto \quad (\vec{s}, \vec{OP} \times \vec{s})$$

- Notation: $(\vec{s}, \vec{OP} \times \vec{s}) = (p_{01}, p_{02}, p_{03}, p_{23}, p_{31}, p_{12})$
- Plücker quadric:

$$(\Pi): \qquad p_{01}p_{23} + p_{02}p_{31} + p_{03}p_{12} = 0$$

From complex cells to sets of lines

- An open cell $\mathcal{C}$ of the complex is homeomorphic to its set of supporting lines
- These supporting lines are the lines that:
  - pierce the balls in $P(\mathcal{C})$
  - are tangent to the balls in $T(\mathcal{C})$
  - miss the balls in $M(\mathcal{C})$



From sets of lines to quadric arrangement

- For a ball B with center (a, b, c) and radius r we consider:

$$Q_B(L) = (p_{23} + p_{02}c - p_{03}b)^2 + (p_{31} + p_{03}a - p_{01}c)^2 + (p_{12} - p_{02}a + p_{01}b)^2 - r^2(p_{01}^2 + p_{02}^2 + p_{03}^2)$$

- Side operator for a ball and a line: $(B, L) \mapsto Q_B(L)$

PARAMETERIZATION OF Q_B

- Π and Q_B are ruled quadric
- "nice" parameterization of $Q_B \cap \Pi$ by $\mathbb{S}^2 \times \mathbb{S}^1$

Results

- The parameterization is a homeomorphism

$$\Longrightarrow \text{ lines tangent to one ball: } \mathbb{S}^2 \times \mathbb{S}^1$$

- Inject the parameterization of $Q_B \cap \Pi$ into $Q_{B'}$

$$\Longrightarrow \text{ lines tangent to two balls: } \mathbb{S}^1 \times \mathbb{S}^1$$

- Extension of Pellegrini's ray-shooting structure [P93]

Future work

- Extend the study of sets of lines :
  - lines tangent to 3 and 4 balls
  - add the conditions of piercing and missing balls
- Connectivity
- Real semi-algebraic projective sets
- Effective computation of intersection of quadrics in $\mathbb{P}^5(\mathbb{R})$