A study of discrete curves based on arithmetic discrete straight lines

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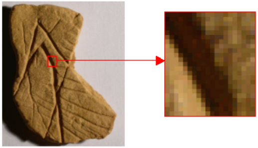
Nancy (FRANCE)

Research

Discrete Geometry

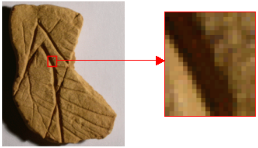


- Digital camera
- Scanners
- Medical Magnetic Resonance Imaging (MRI)
- ...

$\Rightarrow$ Discrete data

Research

Discrete Geometry



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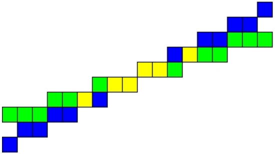


Enclidean theorems are not satisfactory

$\Downarrow$

Discrete Geometry

History

70's : A. Rosenfeld, G. Herman, E. Khalimsky

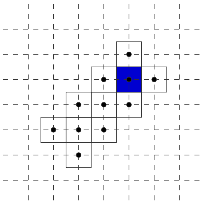
Objective : to define a theoretical framework to transpose in $\mathbb{Z}^n$ the basic notions of Euclidean geometry

Discrete Geometry

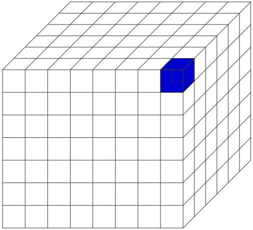
- Grid (representation of data)
- Topology
- Basic Objects (points, straight lines, planes, ...)
- Adapted algorithmic process

Regular Grids

2D discrete space



3D discrete space



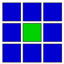
Connectivity

Neighborhood relations

4-connectivity

A and B of $\mathbb{Z}^2$ are 4-neighbour (or 4-adjacent) if :

$$|x_A - x_B| + |y_A - y_B| = 1$$



8-connectivity

A and B of $\mathbb{Z}^2$ are 8-neighbour (or 8-adjacent) if :

$$max(|x_A - x_B|, |y_A - y_B|) = 1$$

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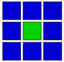

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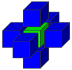
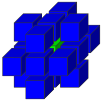
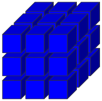


6-connectivity



18-connectivity



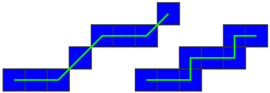
26-connectivity

α-connectivity or α-adjacency

Curves

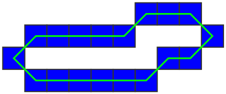
k-Arc

Let $\mathcal{E} = \{p_i\}_{i=0..n}$ be a set of discrete points and a relation of k-adjacency, $\mathcal{E}$ is called a k-arc if for each element p_i of $\mathcal{E}$, p_i has exactly two k-neighbour points in $\mathcal{E}$, excepted p_0 and p_n called extremities of the arc.

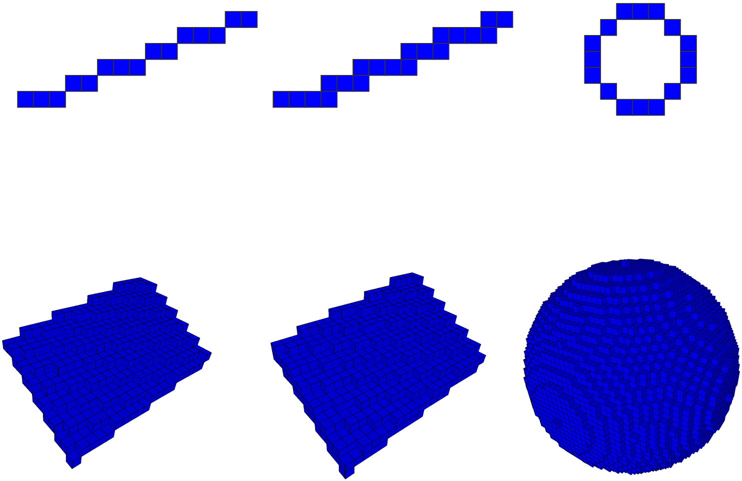


k-Curve

Let $\mathcal{E} = \{p_i\}_{i=0..n}$ be a set of discrete points and a relation of k-adjacency, $\mathcal{E}$ is called a k-curve if $\mathcal{E}$ is a k-arc and $p_0 = p_n$.



Discrete Primitives



Outline of talk

Outline of talk

Arithmetic definition - Réveilles (91)

Arithmetic discrete line

A discrete line with parameters (a, b, μ) and arithmetical thickness ω is defined as the set of integer points (x, y) verifying :

$$\mu \leq ax - by < \mu + \omega$$

- a, b, μ, ω in $\mathbb{Z}$
- $gcd(a, b) = 1$, (b, a) main vector of the line
- noted $\mathcal{D}(a, b, \mu, \omega)$

J.-P. Réveilles,
Géométrie discrète, calcul en nombres entiers et algorithmique.
Thèse d'état, Université Louis Pasteur, Strasbourg, 1991.

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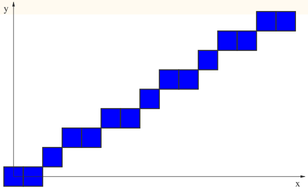
- noted $\mathcal{D}(a, b, \mu, \omega)$

$\omega = \max(|a|, |b|)$: $\mathcal{D}$ is 8-arc and is called a naïve line



$\mathcal{D}(5, 8, -1, 8) : -1 \leq 5x - 8y < 7$

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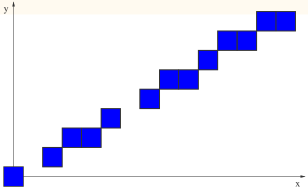
- $gcd(a, b) = 1$, (b, a) main vector of the line
- noted $\mathcal{D}(a, b, \mu, \omega)$

$\omega < \max(|a|, |b|)$: $\mathcal{D}$ is not connected



$\mathcal{D}(5, 8, -1, 7) : -1 \leq 5x - 8y < 6$

Arithmetic definition - Réveilles (91)

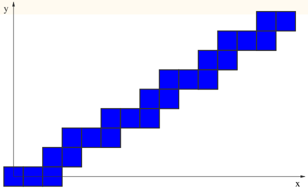
Arithmetic discrete line

A discrete line with parameters (a, b, μ) and arithmetical thickness ω is defined as the set of integer points (x, y) verifying :

$$\mu \leq ax - by < \mu + \omega$$

- $a,\ b,\ \mu,\ \omega$ in $\mathbb{Z}$
- $gcd(a, b) = 1$, (b, a) main vector of the line
- noted $\mathcal{D}(a, b, \mu, \omega)$

$\omega = |a| + |b|$: $\mathcal{D}$ is a 4-arc and is called a standard line



$\mathcal{D}(5, 8, -1, 13) : -1 \leq 5x - 8y < 12$

Arithmetic definition - Réveilles (91)

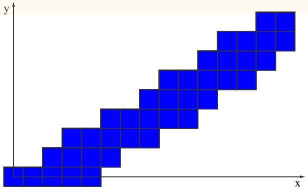
Arithmetic discrete line

A discrete line with parameters (a, b, μ) and arithmetical thickness ω is defined as the set of integer points (x, y) verifying :

$$\mu \leq ax - by < \mu + \omega$$

- a, b, μ, ω in $\mathbb{Z}$
- $gcd(a, b) = 1$, (b, a) main vector of the line
- noted $\mathcal{D}(a, b, \mu, \omega)$

$\omega > |a| + |b|$: $\mathcal{D}$ is called a thick line

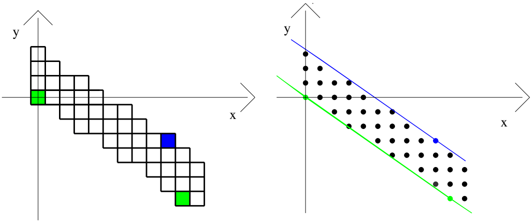


$\mathcal{D}(5, 8, -1, 22) : -1 \leq 5x - 8y < 21$

Leaning lines and points

Definition

- Leaning lines of $\mathcal{D}(a, b, \mu, \omega)$:
  Real lines $ax - by = \mu$ and $ax - by = \mu + \omega - 1$
- Leaning points of $\mathcal{D}(a, b, \mu, \omega)$
- Recognized segment of $\mathcal{D}$: a segment of $\mathcal{D}$ that contains at least 3 leaning points



Recognized segment of $\mathcal{D}(7, -10, 0, 34)$: $0 \leq 7x + 10y < 34$

Construction of a naïve line

Remainder

Definition

Remainder at the point M as a function of $\mathcal{D}(a, b, \mu, \omega)$:

$$r_{\mathcal{D}}(M) = ax_M - by_M$$

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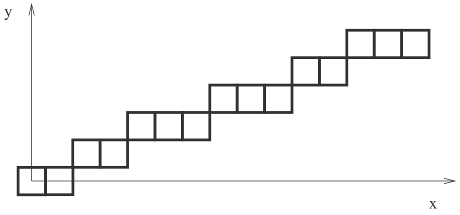

$\mathcal{D}(3, 8, -4, 8)$, $x \in [0, 14]$, $-4 \leq 3x - 8y < 4$

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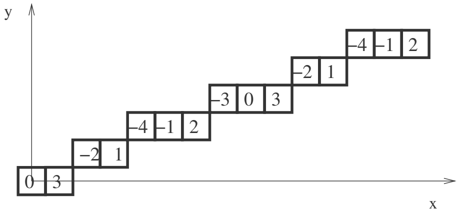

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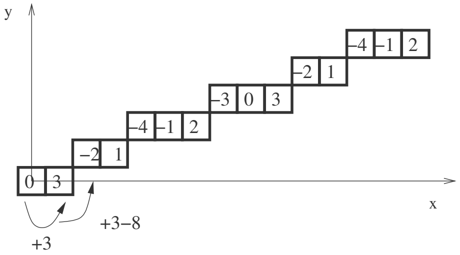

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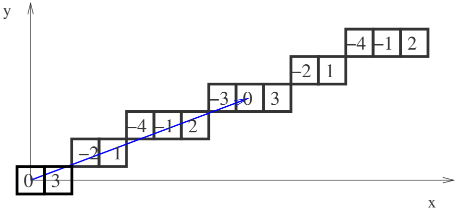
Periodicity

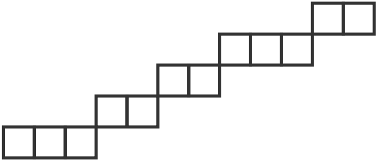


$\mathcal{D}(3, 8, -4, 8)$, $x \in [0, 14]$, $-4 \leq 3x - 8y < 4$

Periodicity

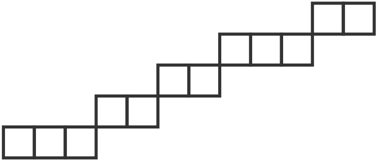
$\mathcal{D}(a, b, \mu, \omega)$ is invariant by the translation $k.(b, a)^T$ with $k \in \mathbb{Z}$

Recognition : the problem



Is that a segment of naïve line ?

Recognition : the problem



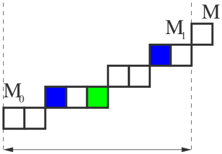
Is that a segment of naïve line ?

Approach :

- Arithmetical
- Incremental

I. DEBLED-RENNESSON, J.-P. REVEILLES,
A linear algorithm for segmentation of digital curves.
IJPRAI, 9(6), 1995.

Linear and incremental recognition algorithm



Growth of a recognized segment of a naïve line

Let be $\mathcal{S} = M_0 M_1$ a recognized segment of $\mathcal{D}(a, b, \mu, max(|a|, |b|))$, M an added point to $\mathcal{S}$, $r_{\mathcal{D}}(M) = ax_M - by_M$:

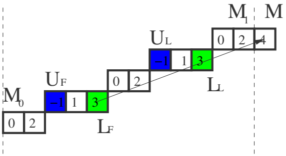
(i) $\mu \leq r_{\mathcal{D}}(M) < \mu + max(|a|, |b|)$: $M \in \mathcal{D}$,
$\mathcal{S} \cup \{M\}$ is a segment of $\mathcal{D}$,

(ii) $r_{\mathcal{D}}(M) = \mu + max(|a|, |b|)$: M is weakly exterior to $\mathcal{D}$,
$\mathcal{S} \cup \{M\}$ is a recognized segment of the naïve line whose slope is given by the vector $L_F M$,

(iii) $r_{\mathcal{D}}(M) = \mu - 1$: M is weakly exterior to $\mathcal{D}$,
$\mathcal{S} \cup \{M\}$ is a recognized segment of the naïve line whose slope is given by the vector $U_F M$,

(iv) $r_{\mathcal{D}}(M) < \mu - 1$ or $r_{\mathcal{D}}(M) > \mu + max(|a|, |b|)$: M is strongly exterior to $\mathcal{D}$,
$\mathcal{S} \cup \{M\}$ is not a segment of a naïve line.

Linear and incremental recognition algorithm



$\mathcal{S} = M_0M_1$ recognized segment of $\mathcal{D}(2, 5, -1, 5)$, $-1 \leq 2x - 5y < 4$
$\mathcal{S} \cup \{M\}$ recognized segment of $\mathcal{D}'(3, 8, -3, 8)$.

Growth of a recognized segment of a naïve line

Let be $\mathcal{S} = M_0M_1$ a recognized segment of $\mathcal{D}(a, b, \mu, max(|a|, |b|))$, M an added point to $\mathcal{S}$, $r_{\mathcal{D}}(M) = ax_M - by_M$:

(*i*) $\mu \leq r_{\mathcal{D}}(M) < \mu + max(|a|, |b|)$: $M \in \mathcal{D}$,
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$\mathcal{S} \cup \{M\}$ is a recognized segment of the naïve line whose slope is given by the vector U_FM,

(*iv*) $r_{\mathcal{D}}(M) < \mu - 1$ or $r_{\mathcal{D}}(M) > \mu + max(|a|, |b|)$: M is strongly exterior to $\mathcal{D}$,
$\mathcal{S} \cup \{M\}$ is not a segment of a naïve line.

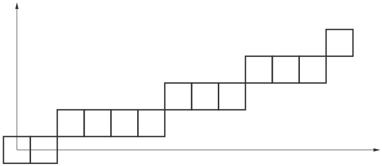
Recognition algorithm of naïve line segments, $0 \leq a \leq b$

Input : C, a sequence of n 8-connected pixels

For each point M of C
    check $r(M)$
    If $r_{\mathcal{D}}(M) = \mu + max(|a|, |b|)$ or $r_{\mathcal{D}}(M) = \mu - 1$ then
        check new characteristics
        update leaning points
    Fsi
    If $r_{\mathcal{D}}(M) < \mu - 1$ ou $r_{\mathcal{D}}(M) > \mu + max(|a|, |b|)$ then
        stop, C is not a naïve line segment
    Fsi

Complexity : $O(n)$

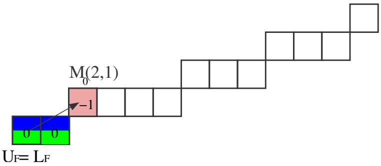
A recognition example



Initialisation : $a = 0,\ b = 1,\ \mu = 0,\ D_0(0, 1, 0, 1)$

$$0 \leq -y < 1$$

A recognition example



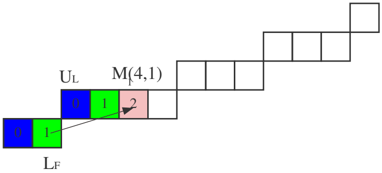
$$a_0 = 0,\ b_0 = 1,\ \mu_0 = 0,\ D_0(0,1,0,1)$$

$$0 \leq -y < 1$$

$$r_0(M_0) = -1 \Rightarrow a_1 = 1,\ b_1 = 2,\ \mu_1 = 0,\ D_1(1,2,0,2)$$

$$0 \leq x - 2y < 2$$

A recognition example



$$a_1 = 1,\ b_1 = 2,\ \mu_1 = 0,\ D_1(1,2,0,2)$$

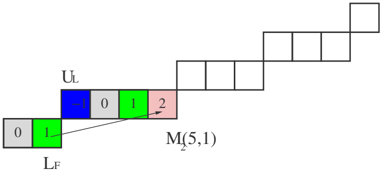
$$0 \le x - 2y < 2$$

$$r_1(M_1) = 2 \Rightarrow a_2 = 1,\ b_2 = 3,\ \mu_2 = -1,\ D_2(1,3,-1,3)$$

$$-1 \le x - 3y < 2$$

Discrete Line
Arithmetic definition
Recognition
Applications
Blurred segments
Definitions
Recognition
Applications
Conclusion

A recognition example



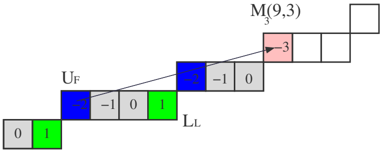
$$a_2 = 1,\ b_2 = 3,\ \mu_2 = -1,\ D_2(1, 3, -1, 3)$$

$$-1 \leq x - 3y < 2$$

$$r_2(M_2) = 2 \Rightarrow a_3 = 1,\ b_3 = 4,\ \mu_3 = -2,\ D_3(1, 4, -2, 4)$$

$$-2 \leq x - 4y < 2$$

A recognition example



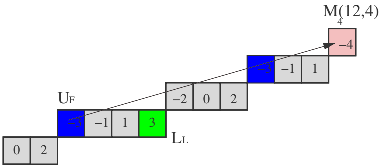
$$a_3 = 1,\ b_3 = 4,\ \mu_3 = -2,\ D_3(1, 4, -2, 4)$$

$$-2 \leq x - 4y < 2$$

$$r3(M_3) = -3 \Rightarrow a_4 = 2,\ b_4 = 7,\ \mu_4 = -3,\ D_4(2, 7, -3, 7)$$

$$-3 \leq 2x - 7y < 4$$

A recognition example



$$a_4 = 2,\ b_4 = 7,\ \mu_4 = -3,\ D_4(2, 7, -3, 7)$$

$$-3 \le 2x - 7y < 4$$

$$r4(M_4) = -4 \Rightarrow a_5 = 3,\ b_5 = 10,\ \mu_5 = -4,\ D_5(3, 10, -4, 10)$$

$$-4 \le 3x - 10y < 6$$

Applications of the recognition algorithm

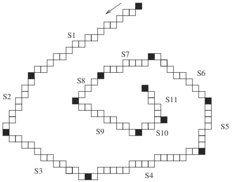
1. Segmentation and polygonalization of 2D discrete curves
   - Minimal number of segments, convexity, ...
2. Extraction of geometrical parameters on 2D discrete curves
   - Length, curvature
3. 3D discrete curves
   - Recognition
   - Segmentation
   - Length, curvature

Segmentation of discrete curves

First algorithm

Objective : Maximal segmentation of a 2D discrete curve

To decompose a discrete 2D curve into naïve discrete line segments of maximal length by starting at a given point of the curve



Symmetries of the naïve discrete lines
Incremental recognition algorithm

⇓

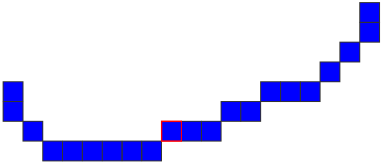
Linear algorithm of curve segmentation

Segmentation of discrete curves

Fundamental segments of a discrete curve

Fundamental segment of a discrete curve

Let $\mathcal{C}$ be a discrete curve, a segment of a naïve discrete line is said fundamental (or maximal) if it cannot be extended at the right and left hand sides on $\mathcal{C}$.



Segmentation of discrete curves

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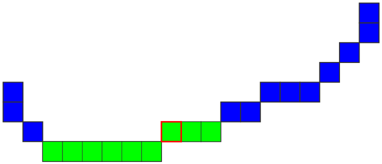

Segmentation of discrete curves

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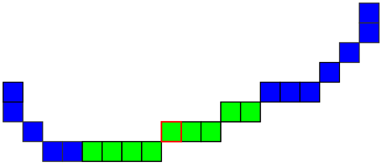
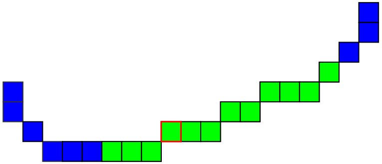


Segmentation of discrete curves

Fundamental segments of a discrete curve

Fundamental segment of a discrete curve

Let $\mathcal{C}$ be a discrete curve, a segment of a naïve discrete line is said fundamental (or maximal) if it cannot be extended at the right and left hand sides on $\mathcal{C}$.



Segmentation of discrete curves

Fundamental segments of a discrete curve

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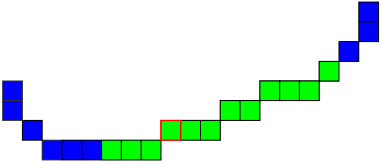

Algorithm to compute the sequence of all fundamental segments of a discrete curve of n points

(F. Feschet, L. Tougne 99)

Segmentation of discrete curves

Algorithm to compute the sequence of all fundamental segments of a discrete curve

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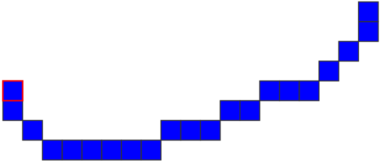
Complexity : $O(n)$



F. FESCHET, L. TOUGNE,
Optimal Time Computation of the Tangent of a Discrete Curve : Application to the Curvature.
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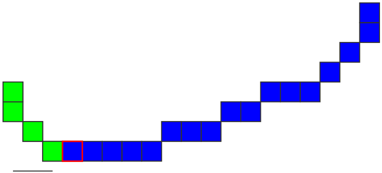

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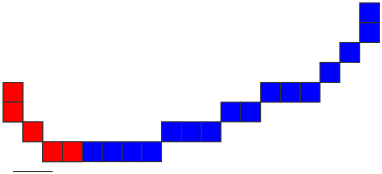

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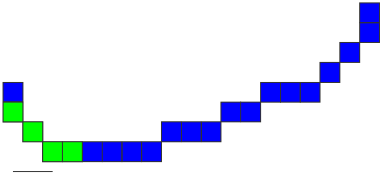

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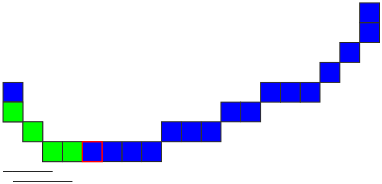

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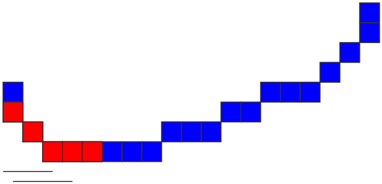

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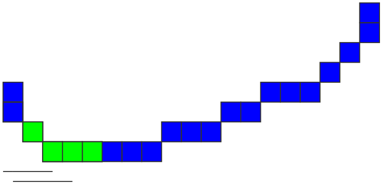

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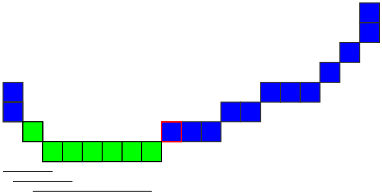

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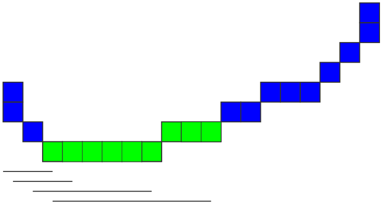

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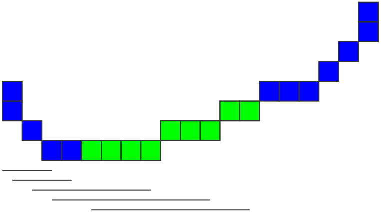

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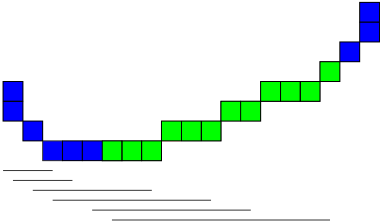

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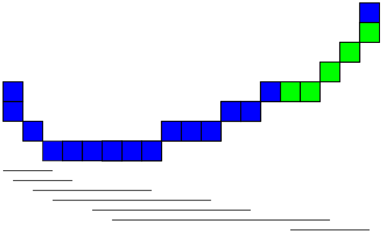

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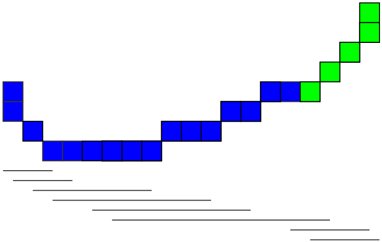

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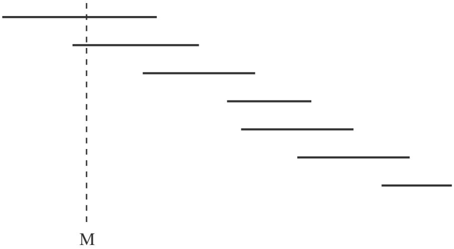
Segmentation of discrete curves

Using fundamental segments of a discrete curve

All maximal segmentations of a given curve can be obtained by using its sequence of fundamental segments.



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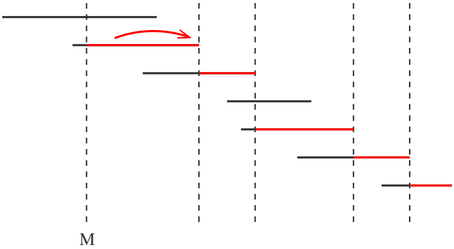

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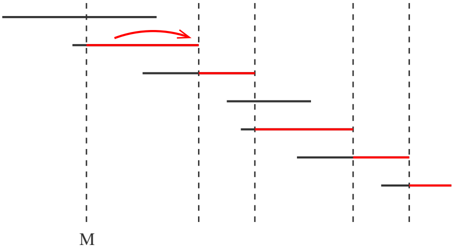


For all pair of maximal segmentations of a given closed curve, the difference between their number of segments is 0 or 1.

F. FESCHET, L. TOUGNE,
On the min DSS problem of closed discrete curves.
Discrete Applied Mathematics 151(1-3) : 138-153, 2005.

Polygonalization of 2D discrete curves

with Hélène Dörksen-Reiter

Objectives

- *Reversible* polygonalization
- To keep the *convexity properties* of the discrete curve
- *All vertices* of the polygonalization are *in* $\mathbb{Z}^2$

Polygonalization of 2D discrete curves

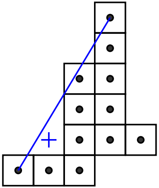
Convexity

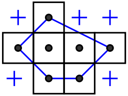
Discrete convexity

A discrete object O is convex iff its convex (Euclidean) hull does not contain any discrete point of the complementary of O.

C. E. KIM, A. ROSENFELD,
Digital Straightness and Convexity .
STOC : 80-89, 1981.

C. E. KIM, J. SLANSKY,
Digital and cellular convexity.
Pattern Recognition 15(5) : 359-367, 1982.



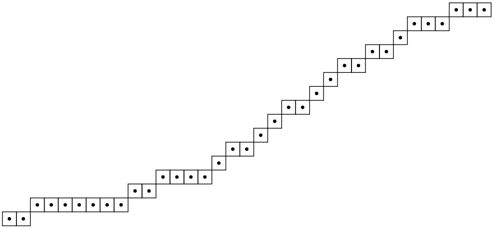


Polygonalization of 2D discrete curves

Convexity

Curve of the boundary and convexity

In the first octant, a 8-curve C of the boundary of O is said convex (resp.concave) if the fundamental segments of C have strictly increasing (resp. decreasing) slopes.



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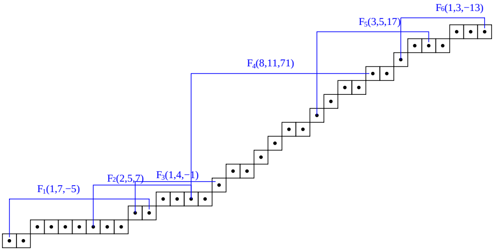
In the first octant, a 8-curve C of the boundary of O is said convex (resp.concave) if the fundamental segments of C have strictly increasing (resp. decreasing) slopes.

C is convex $\Leftrightarrow$ there is no discrete point between C and its lower convex hull.



$p_1 = 0.14 < p_2 = 0.4$, maximal convex part

$p_2 = 0.4 > p_3 = 0.25$, maximal concave part

$p_3 = 0.25 < p_4 = 0.72$, maximal convex part

$p_4 = 0.72 > p_5 = 0.6 > p_6 = 0.33$, maximal concave part

Polygonalization of 2D discrete curves

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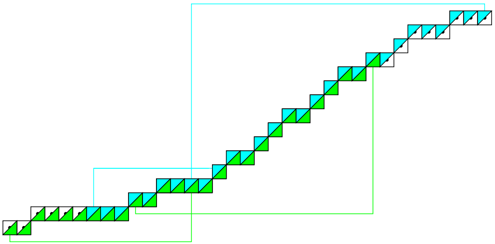

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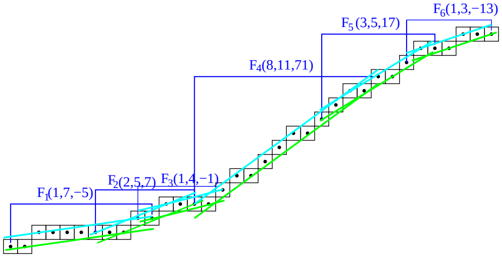
Polygonalization of 2D discrete curves

Keeping the succession of convex and concave parts

Lower (Upper) fundamental polygonal representation of C

Polygonal curve whose vertices are the intersection points of the successive lower (resp. upper) leaning lines of the fundamental segments of C.



Using of the leaning lines of the fundamental segments

Polygonalization of 2D discrete curves

Keeping the succession of convex and concave parts

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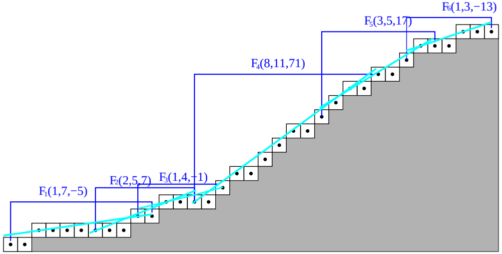

Keeping the succession of convex and concave parts

Reversibility : a digitization of the obtained polygonal curve corresponds to the discrete curve

Polygonalization of 2D discrete curves

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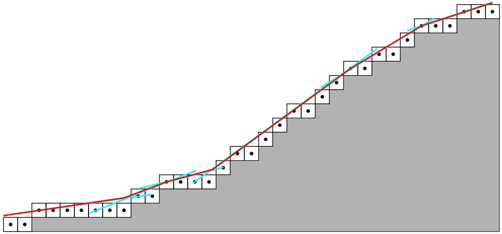

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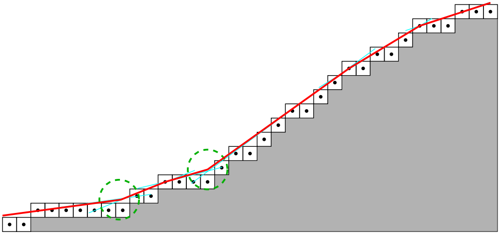


Keeping the succession of convex and concave parts

Reversibility : a digitization of the obtained polygonal curve corresponds to the discrete curve

The vertices of the obtained polygonalization are not always points of Z^2

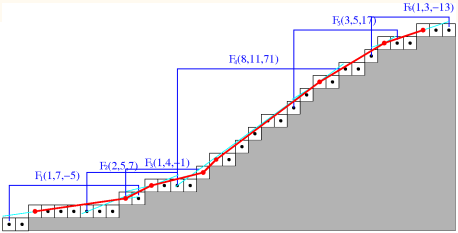
Polygonalization of 2D discrete curves

Keeping the succession of convex and concave parts

Results

- Linear algorithm of polygonalisation
  - reversible,
  - keeping the convexity/concavity parts of the discrete curve
- Identification of situations where a polygonal decomposition under the two previous conditions
  - with vertices in $\mathbb{Z}^2$,

  is not possible.



H. DÖRKSEN-REITER, I. DEBLED-RENNESSON,

Convex and concave parts of digital curves, Computational Imaging and Vision, 2005.
A Linear algorithm for polygonal representations of digital sets, IWCIA, 2006.

3D discrete lines

Definition

3D discrete line

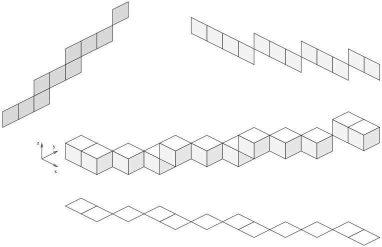
A 3D discrete line, noted $\mathcal{D}(a, b, c, \mu, \mu', e, e')$, whose main vector is (a, b, c), with $(a, b, c) \in \mathbb{Z}^3$, and $a \geq b \geq c$ is the set of points (x, y, z) of $\mathbb{Z}^3$ verifying :

$$\mathcal{D} \begin{cases} \mu \leq cx - az < \mu + e & (1) \\ \mu' \leq bx - ay < \mu' + e' & (2) \end{cases}$$

with $\mu, \mu', e, e' \in \mathbb{Z}$. e and e' are the arithmetical thickness of $\mathcal{D}$.

Naïve line : $e = e' = a$

$$\begin{cases} 0 \leq 3x - 10z < 10 \\ 0 \leq 7x - 10y < 10 \end{cases}$$



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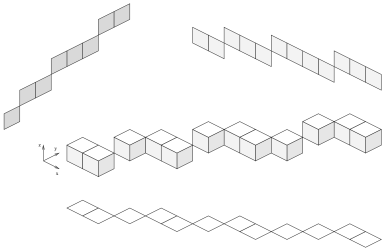
Naïve line : $e = e' = a$

$$\begin{cases} -5 \leq 3x - 10z < 5 \\ 0 \leq 7x - 10y < 10 \end{cases}$$



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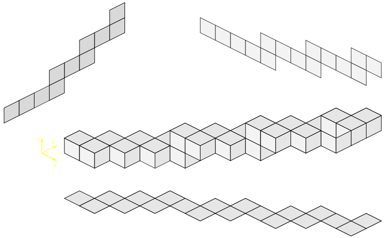
6-connected line :
$e \geq a + c$ et $e' \geq a + b$

$$\begin{cases} 0 \leq 3x - 10z < 13 \\ -9 \leq 7x - 10y < 8 \end{cases}$$



3D discrete lines

Algorithm for 3D naïve line segment recognition

Property : A 3D naïve line is bijectively projected into two coordinates planes as two 2D naïve lines

Input : S, a 26-connected sequence of n voxels to be analysed

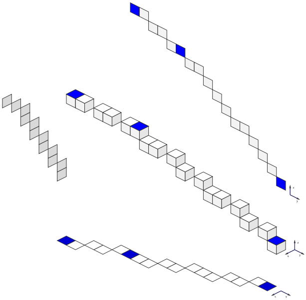
- **If** the voxels of S may not be bijectively projected on at least two orthogonal planes in order to create two curves of pixels C_1 and C_2, S is not a 3D naïve line segment,
- **Else**, apply the algorithm of 2D naïve line segment recognition on C_1 and C_2,

    If C_1 and C_2 are 2 naïve line segments, **then** S is a 3D naïve line segment

    Else S is not a 3D naïve line segment

Complexity : $O(n)$

$\Rightarrow$ Linear segmentation algorithm



Segment 1 of main vector $(2, -5, 4)$

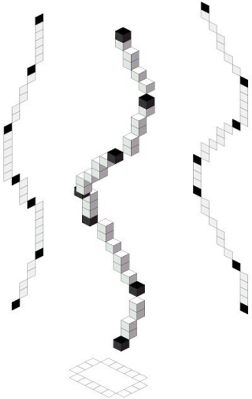
$$\begin{cases} -4 \leq -4x - 5z < 1 \\ -2 \leq -2x - 5y < 3 \end{cases}$$

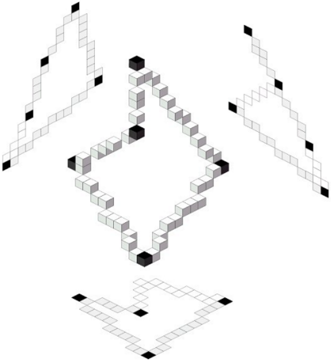
Segment 2 of main vector $(1, -2, 1)$

$$\begin{cases} 0 \leq x - 2z < 2 \\ 0 \leq x - 2y < 2 \end{cases}$$

Segmentation of 3D discrete curves

Examples





Length estimation algorithm

Input : S, a 26-connected sequence of voxels to be analysed

Output : The estimated length of S

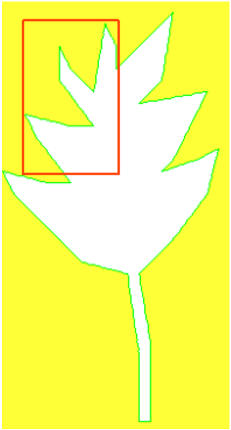
- Compute a segmentation of S
- $P = \{S_i\}_{i=0..n}$, the polyline returned by the segmentation
- Return $\sum_{i=0}^{n} l(S_i)$,
  where $l(S_i)$ denotes the Euclidean length of S_i

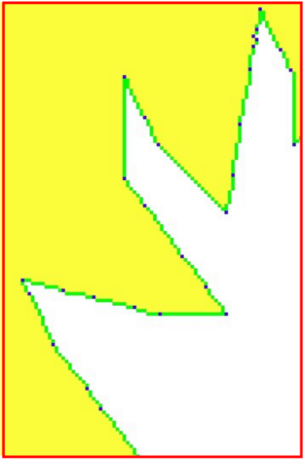
D. COEURJOLLY, I. DEBLED-RENNESSON, O. TEYTAUD
Segmentation and Length Estimation of 3D Discrete Curves.
Digital and Image Geometry, LNCS 2243 : 299-317, 2000.

Outline of talk

Limitation of the existing tools of discrete geometry

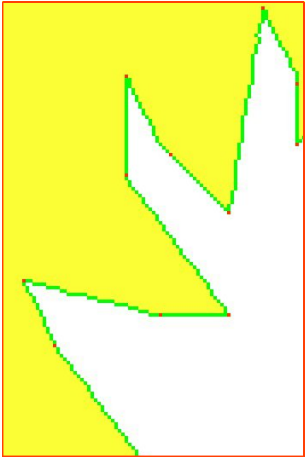
Limitation of the segmentation algorithm (naïve lines).





Objectives

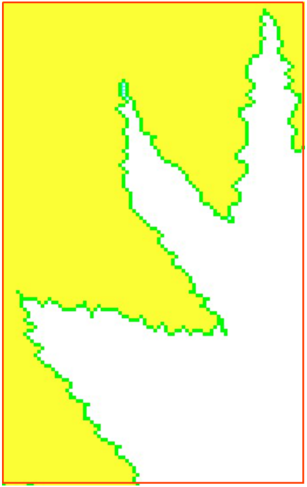
What we want to obtain ...



Objectives

What we want to obtain ...

- ▷ Also for very noisy curves



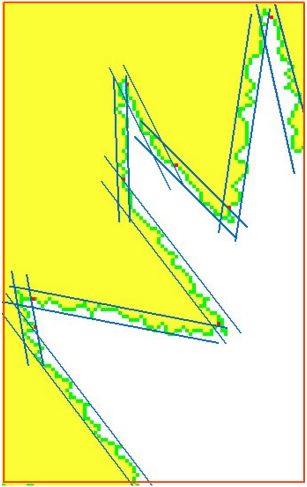
Objectives

What we want to obtain ...

- ▷ Also for very noisy curves

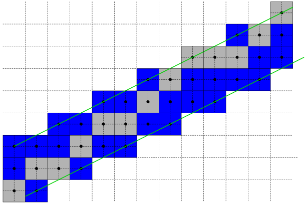
General idea

- ▷ Frame the curve with thick discrete lines for a given maximal thickness



Arithmetic blurred segments

Bounding lines



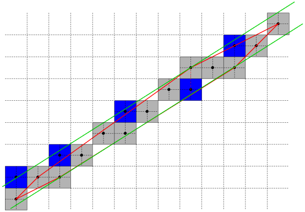
$\mathcal{D}(1, 2, -4, 6)$, bounding line of the sequence of grey points

Bounding line

Let be $\mathcal{S}f$ a sequence of 8-connected points.
A discrete line $\mathcal{D}(a, b, \mu, \omega)$ is said bounding for $\mathcal{S}f$ if all the points of $\mathcal{S}f$ belong to $\mathcal{D}$.

Arithmetic blurred segments

Geometrical approach
With Jocelyne Rouyer-Dégli and Fabien Feschet



$\mathcal{D}(5, 8, -8, 11)$, optimal bounding line (width $\frac{10}{8} = 1.25$) of the sequence of grey points

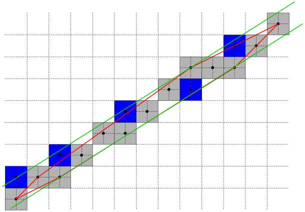
Optimal bounding line

A bounding line $\mathcal{D}(a, b, \mu, \omega)$ of $\mathcal{S}f$ is said optimal if its vertical width is equal to the vertical width of the convex hull of $\mathcal{S}f$.

▷ Vertical width of $\mathcal{D}(a, b, \mu, \omega)$: $\frac{\omega - 1}{max(|a|, |b|)}$

Arithmetic blurred segments

Geometrical approach
With Jocelyne Rouyer-Dégli and Fabien Feschet



The sequence of grey points is a blurred segment of width 2

Optimal bounding line

A bounding line $\mathcal{D}(a, b, \mu, \omega)$ of $\mathcal{S}f$ is said optimal if its vertical width is equal to the vertical width of the convex hull of $\mathcal{S}f$.

▷ Vertical width of $\mathcal{D}(a, b, \mu, \omega)$: $\frac{\omega - 1}{max(|a|, |b|)}$

Blurred segment of width ν

$\mathcal{S}f$ is a blurred segment of width ν if the vertical width of its optimal bounding line is lower or equal to ν.

Blurred segments recognition

The principle

Computation of the vertical width of the convex hull of $\mathcal{S}f$

- Similar to the Rotating Calipers [HouleToussaint88]
- Extremal positions
- Incremental and linear computation of the convex hull
  - Melkman's algorithm

M.E. Houle, G.T. Toussaint,
Computing the width of a set.
PAMI, 10(5) :761–765, 1988.

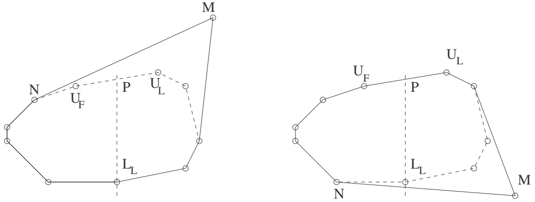
A. Melkman,
On-line Construction of the Convex Hull of a Simple Polygon.
Information Processing Letters, 25 :11–12, 1987.

Blurred segments recognition

The principle

Adding a point $M(x, y)$ to a blurred segment $\mathcal{S}_f = \{(x_i, y_i), 0 \leq i < n\}$ with $\mathcal{D}(a, b, \mu, \omega)$ its optimal bounding line in the first octant $x > x_{n-1}$.
3 cases are possible :

- M belongs to $\mathcal{D}$,
  $\mathcal{S}'f = \mathcal{S}f \cup M$ is a blurred segment with $\mathcal{D}$ as optimal bounding line,
- M is above $\mathcal{D}$,
- M is below $\mathcal{D}$.

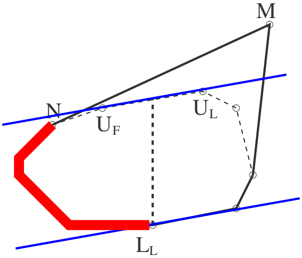


Blurred segments recognition

The principle

Adding a point M to a blurred segment $\mathcal{S}f$ with $\mathcal{D}(a, b, \mu, \omega)$ its optimal bounding line :

M is above $\mathcal{D}$ and the vertical width of $\mathcal{S}f$ is obtained at the point L_L.

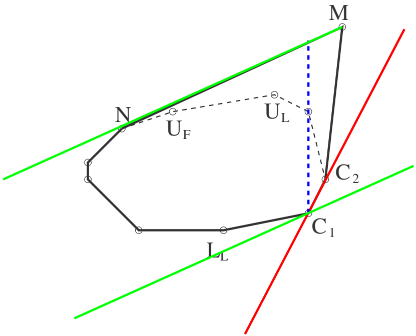


- Objective : to find the optimal bounding line $\mathcal{D}'$ of $\mathcal{S}'f = \mathcal{S}f \cup M$.
- Property : the vertical width in a convex is a concave function and the maximum is located inside the convex.
- To find the location of the maximum in the new convex
  - $\Rightarrow$ necessarily at the right of L_L

Blurred segments recognition

The principle

Adding a point M to a blurred segment $\mathcal{S}f$ with $\mathcal{D}(a, b, \mu, \omega)$ as optimal bounding line :



- Test the vertices of the convex hull located at the right of L_L
- Test : slope of $[C_1C_2] >$ slope of $[NM]$? TRUE $\Rightarrow$ STOP

$\Rightarrow$ The vertical width of the convex is obtained at C_1

$\Rightarrow$ The slope of the optimal bounding line of $\mathcal{S}f \cup M$ is $\overrightarrow{NM}$ and C_1 is a lower leaning point

Blurred segments recognition

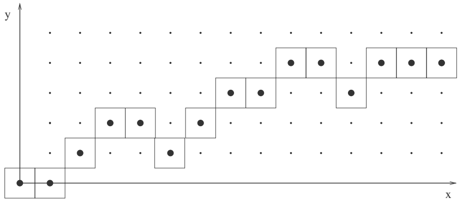
The algorithm in the first octant

Input : S an 8-connected sequence of integer points, ν a real value
Output : *isSegment* a boolean value, a, b, μ, ω integers

Initialization : $isSegment = true$, $a = 0$, $b = 1$, $\omega = b$, $\mu = 0$, $M = (x_0, y_0)$
while (S is not entirely scanned and *isSegment*)
  M = next point of S ;
  add M to the upper and lower convex hulls of the scanned part of S ;
  $r = ax_M - by_M$;
  if $(r = \mu)$ **then** $U_L = M$;
  if $(r = \mu + \omega - 1)$ **then** $L_L = M$;
  if $(r \leq \mu - 1)$ **then**
    $U_L = M$;
    Let N the point before M in the upper convex hull,
    $a_0 = y_M - y_N$, $b_0 = x_M - x_N$,
    $a = \frac{a_0}{gcd(a_0, b_0)}$, $b = \frac{b_0}{gcd(a_0, b_0)}$, $\mu = ax_M - by_M$;
    Find the first point C in the lower part of the convex hull starting at L_L,
    such that : slope of $[C, Cnext] > \frac{a}{b}$;
    $L_L = C$;
    $\omega = ax_{L_L} - by_{L_L} - \mu + 1$;
  else
    if $(r \geq \mu + \omega - 1)$ **then** *symmetrical case*
  end if
  $isSegment = \frac{\omega - 1}{b} \leq \nu$;
end

Blurred segments recognition

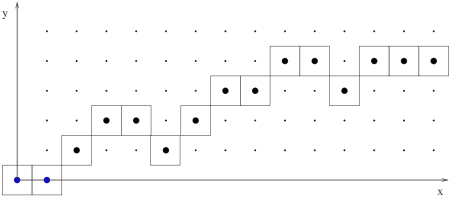
Example



Sequence of pixels to recognize, $\nu = 2$

Blurred segments recognition

Example

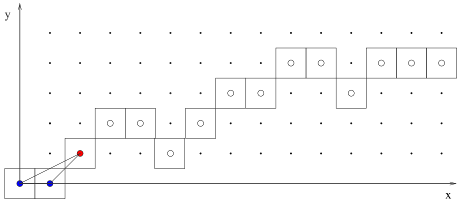


Sequence of pixels to recognize, $\nu = 2$

$$\mathcal{D}_0(0,1,0,1) : 0 \leq -y < 1$$

Blurred segments recognition

Example



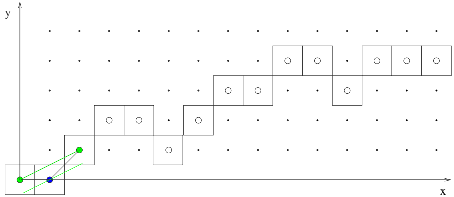
Sequence of pixels to recognize, $\nu = 2$

$$\mathcal{D}_0(0,1,0,1) : 0 \leq -y < 1$$

Adding of the point M_3, $r_{\mathcal{D}_0}(M_3) = -1$

Blurred segments recognition

Example



Sequence of pixels to recognize, $\nu = 2$

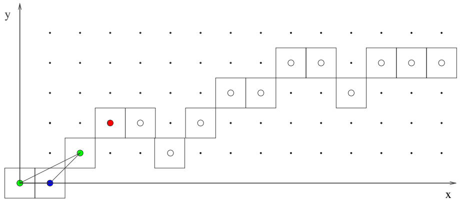
$$\mathcal{D}_0(0, 1, 0, 1) : 0 \leq -y < 1$$

Adding of the point M_3, $r_{\mathcal{D}_0}(M_3) = -1$

$$\mathcal{D}_1(1, 2, 0, 2) : 0 \leq x - 2y < 2,\ d_v = 0.5$$

Blurred segments recognition

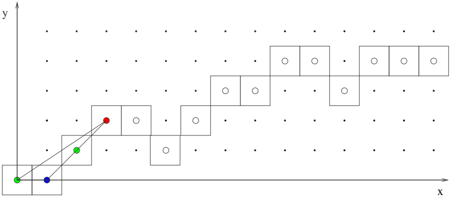
An example



$$\mathcal{D}_1(1,2,0,2) : 0 \leq x - 2y < 2$$

Blurred segments recognition

An example

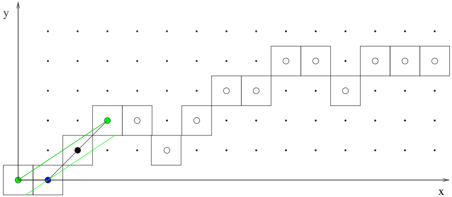


$$\mathcal{D}_1(1,2,0,2) : 0 \leq x - 2y < 2$$

Adding of the point M_4, $r_{\mathcal{D}_1}(M_4) = -1$

Blurred segments recognition

An example



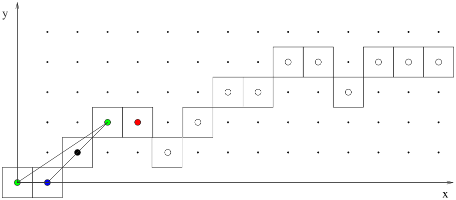
$$\mathcal{D}_1(1,2,0,2) : 0 \leq x - 2y < 2$$

Adding of the point M_4, $r_{\mathcal{D}_1}(M_4) = -1$

$$\mathcal{D}_2(2,3,0,3) : 0 \leq 2x - 3y < 3, \ d_v \simeq 0.66$$

Blurred segments recognition

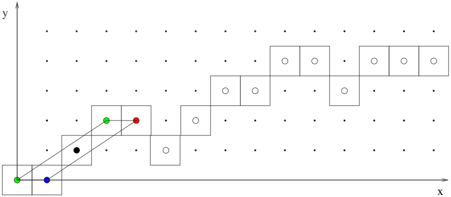
An example



$$\mathcal{D}_2(2,3,0,3) : 0 \leq 2x - 3y < 3$$

Blurred segments recognition

An example

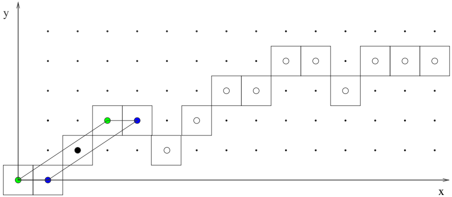


$$\mathcal{D}_2(2,3,0,3) : 0 \leq 2x - 3y < 3$$

Adding of the point M_5, $r_{\mathcal{D}_2}(M_5) = 2$

Blurred segments recognition

An example

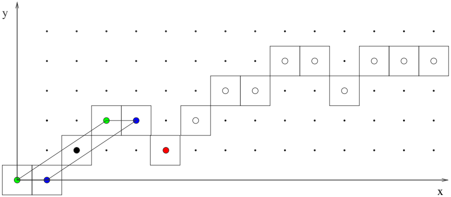


$$\mathcal{D}_2(2,3,0,3) : 0 \leq 2x - 3y < 3$$

Adding of the point M_5, $r_{\mathcal{D}_2}(M_5) = 2$

Blurred segments recognition

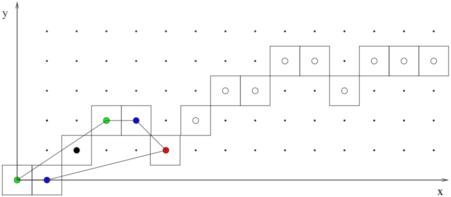
An example



$$\mathcal{D}_2(2,3,0,3) : 0 \leq 2x - 3y < 3$$

Blurred segments recognition

An example

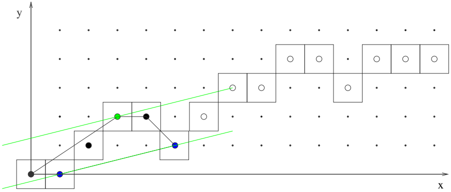


$$\mathcal{D}_2(2,3,0,3) : 0 \leq 2x - 3y < 3$$

Adding of the point M_6, $r_{\mathcal{D}_2}(M_6) = 7$

Blurred segments recognition

An example



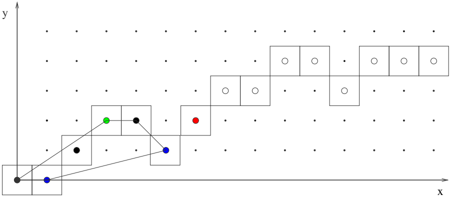
$$\mathcal{D}_2(2,3,0,3) : 0 \leq 2x - 3y < 3$$

Adding of the point M_6, $r_{\mathcal{D}_2}(M_6) = 7$

$$\mathcal{D}_3(1,4,-5,7) : -5 \leq x - 4y < 2,\ d_v = 1.5$$

Blurred segments recognition

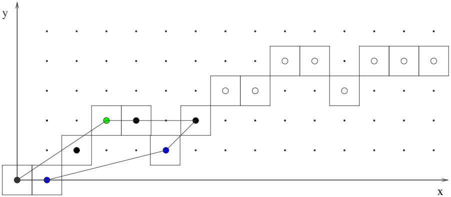
An example



$$\mathcal{D}_3(1, 4, -5, 7) : -5 \leq x - 4y < 2$$

Blurred segments recognition

An example

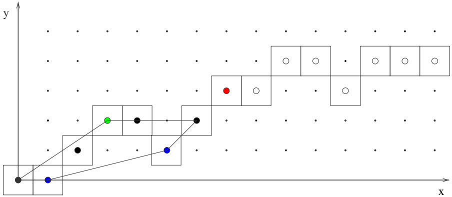


$$\mathcal{D}_3(1, 4, -5, 7) : -5 \leq x - 4y < 2$$

Adding of the point M_7, $r_{\mathcal{D}_3}(M_7) = 2$

Blurred segments recognition

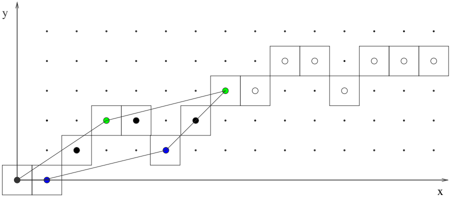
An example



$$\mathcal{D}_3(1, 4, -5, 7) : -5 \leq x - 4y < 2$$

Blurred segments recognition

An example

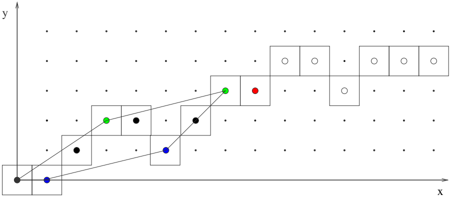


$$\mathcal{D}_3(1, 4, -5, 7) : -5 \leq x - 4y < 2$$

Adding of the point M_8, $r_{\mathcal{D}_3}(M_8) = -5$

Blurred segments recognition

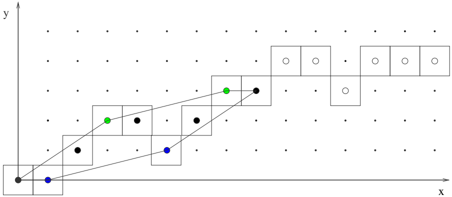
An example



$$\mathcal{D}_3(1, 4, -5, 7) : -5 \leq x - 4y < 2$$

Blurred segments recognition

An example

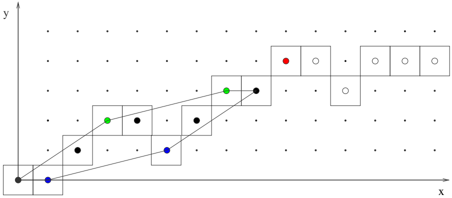


$$\mathcal{D}_3(1,4,-5,7) : -5 \leq x - 4y < 2$$

Adding of the point M_9, $r_{\mathcal{D}_3}(M_9) = -4$

Blurred segments recognition

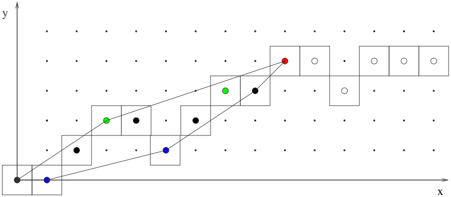
An example



$$\mathcal{D}_3(1, 4, -5, 7) : -5 \leq x - 4y < 2$$

Blurred segments recognition

An example

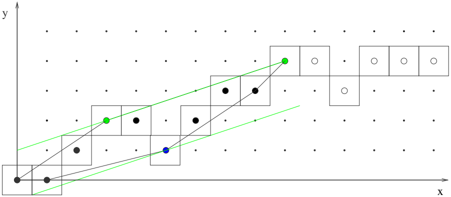


$$\mathcal{D}_3(1,4,-5,7) : -5 \leq x - 4y < 2$$

Adding of the point M_{10}, $r_{\mathcal{D}_3}(M_{10}) = -7$

Blurred segments recognition

An example



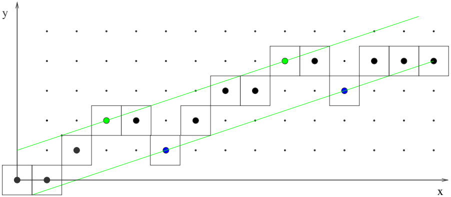
$$\mathcal{D}_3(1, 4, -5, 7) : -5 \leq x - 4y < 2$$

Adding of the point M_{10}, $r_{\mathcal{D}_3}(M_{10}) = -7$

$$\mathcal{D}_4(1, 3, -3, 6) : -3 \leq x - 3y < 3,\ d_v \simeq 1.66$$

Blurred segments recognition

An example



Blurred segment of width 2 with $\mathcal{D}_4(1, 3, -3, 6)$ optimal bounding line

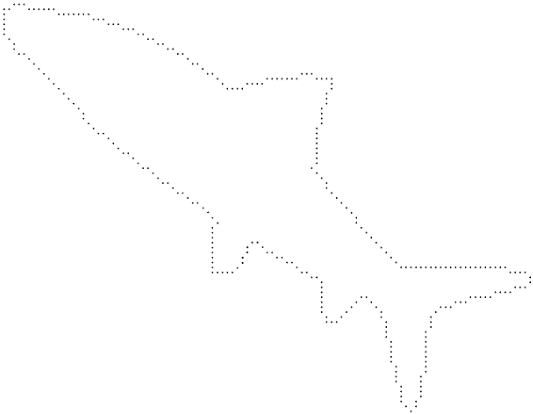
Blurred segment recognition

The algorithm

- Incremental and linear algorithm
  - Tests in a limited part of the convex hull
- Direct extension to the sequences of non connected points
  - Sequence of ordered points

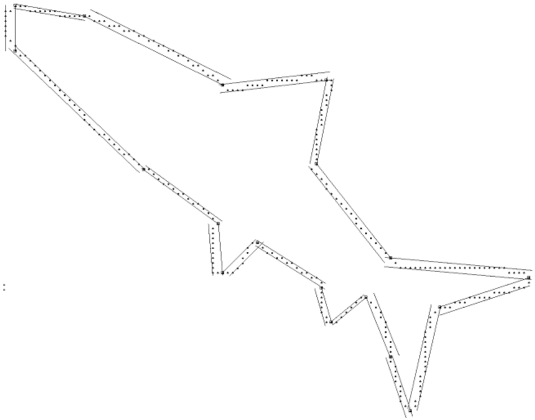
I. DEBLED-RENNESSON, F. FESCHET, J. ROUYER-DEGLI,
Optimal blurred segments decomposition of noisy shapes in linear time.
Computers and Graphics, 30(1), 2006.

Discrete curves segmentation



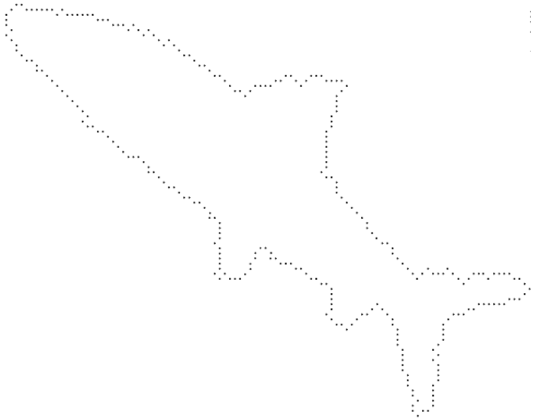
Maximal segmentation of width 2

Discrete curves segmentation



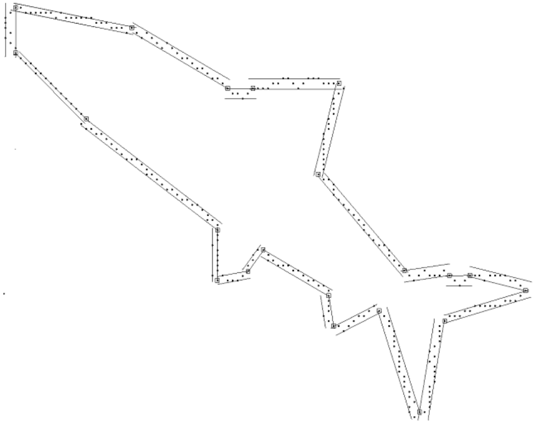
Maximal segmentation of width 2

Discrete curves segmentation



Maximal segmentation of width 2

Discrete curves segmentation



Maximal segmentation of width 2

Applications

1. Segmentation of noisy discrete curves
   - Use in Image Analysis : Polygonal approximation without parameter

2. Estimation of geometrical parameters on noisy discrete curves

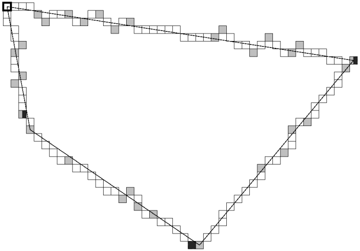
3. Study of 3D noisy curves
   - 3D Blurred Segments

I. DEBLED-RENNESSON, S. TABBONE, L. WENDLING,
Multiorder polygonal approximation of digital curves.
Electronic Letters on Computer Vision and Image Analysis, 5(2) :98-110, August 2005.

Estimation of geometrical parameters on noisy discrete curves

Use of blurred segments

- Length of a noisy discrete curve
  - Use of the polygonal approximation of the curve for a given width



I. DEBLED-RENNESSON,
Estimation of tangents to a noisy discrete curve, Vision Geometry XII, SPIE, 2004.

J-P. SALMON, I. DEBLED-RENNESSON, L. WENDLING,
A new method to detect arcs and segments from curvature profiles, ICPR 2006.

Estimation of geometrical parameters on noisy discrete curves

Use of blurred segments

- Length of a noisy discrete curve
  - Use of the polygonal approximation of the curve for a given width
- Discrete tangent of width ν
  - Symmetric growth of a blurred segment
  - For $\nu = 1$, definition of Anne Vialard (96)

I. DEBLED-RENNESSON,
Estimation of tangents to a noisy discrete curve, Vision Geometry XII, SPIE, 2004.

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A new method to detect arcs and segments from curvature profiles, ICPR 2006.

Estimation of geometrical parameters on noisy discrete curves

Use of blurred segments

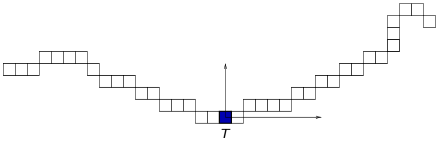
- Length of a noisy discrete curve
  - Use of the polygonal approximation of the curve for a given width
- Discrete tangent of width ν
  - Symmetric growth of a blurred segment
  - For $\nu = 1$, definition of Anne Vialard (96)
- Curvature at each point of a noisy discrete curve
  - Application to the detection of arcs and segments in technical documents

I. DEBLED-RENNESSON,
Estimation of tangents to a noisy discrete curve, Vision Geometry XII, SPIE, 2004.

J-P. SALMON, I. DEBLED-RENNESSON, L. WENDLING,
A new method to detect arcs and segments from curvature profiles, ICPR 2006.

Curvature of width ν

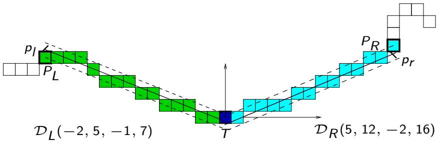
Use of blurred segments



Principle (D. Coeurjolly, 02)

Curvature of width ν

Use of blurred segments



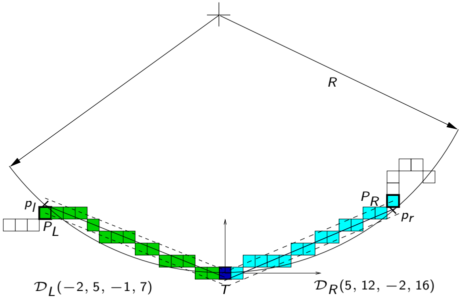
Example of computation of the curvature of width 1.3 at the point T

Principle (D. Coeurjolly, 02)

- Calculate the width ν discrete half-tangents at the right and at the left of T
  - ▷ Bounding lines $\mathcal{D}_R$ and $\mathcal{D}_L$ $\Rightarrow$ real points p_R and p_L

Curvature of width ν

Use of blurred segments



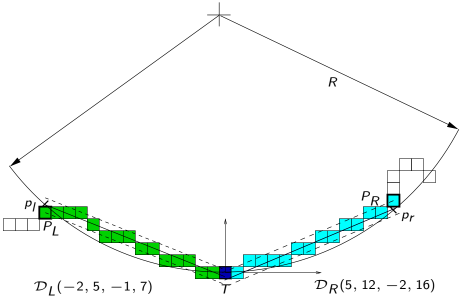
Example of computation of the curvature of width 1.3 at the point T

Principle (D. Coeurjolly, 02)

- Calculate the width ν discrete half-tangents at the right and at the left of T
  - ▷ Bounding lines $\mathcal{D}_R$ and $\mathcal{D}_L$ $\Rightarrow$ real points p_R and p_L
- Calculate the circumcircle of the triangle (p_l, T, p_r)
  - ▷ $C_\nu(T) = \frac{S}{R_\nu(T)}$ with $S = sign(det(\overrightarrow{Tp_r}, \overrightarrow{Tp_l}))$

Curvature of width ν

Use of blurred segments



Example of computation of the curvature of width 1.3 at the point T

Principle (D. Coeurjolly, 02)

- Calculate the width ν discrete half-tangents at the right and at the left of T
  - Bounding lines $\mathcal{D}_R$ and $\mathcal{D}_L \Rightarrow$ real points p_R and p_L
- Calculate the circumcircle of the triangle (p_l, T, p_r)
  - $C_\nu(T) = \frac{S}{R_\nu(T)}$ with $S = sign(det(\overrightarrow{Tp_r}, \overrightarrow{Tp_l}))$
- Calculate the curvature at each point of a discrete curve of n points : $O(n^2)$

Curvature of width ν

Improvement of the calculation of the curvature at each point of a discrete curve of n points

With Thanh Phuong Nguyen

Principle

- Extension of the notion of fundamental segment in a discrete curve
  - Width ν fundamental blurred segment
  - Computation of the sequence of the fundamental blurred segments of a discrete curve C for a given width ν

    Complexity $0(nlog^2 n)$ *(L. Buzer 05) et (M.H. Overmars, J. van Leeuwen 81)*

T.P. NGUYEN, I. DEBLED-RENNESSON,
Curvature estimation in noisy curves, CAIP, 2007.

J-P. SALMON, I. DEBLED-RENNESSON, L. WENDLING,
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- Complexity of the method : $0(nlog^2 n)$

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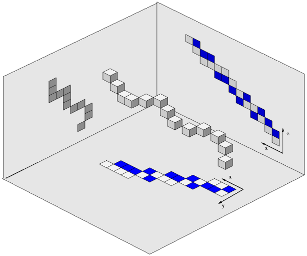
Extension to 3D noisy curves

With Franck Rapaport and Thanh Phuong Nguyen

- 3D blurred segment of width ν
  - ▷ Two projections in the coordinate planes are 2D blurred segments of width ν



$\mathcal{D}_{3D}(45, 27, 20, -45, -81, 90, 90)$ bounding line of the grey points

Extension to 3D noisy curves

With Franck Rapaport and Thanh Phuong Nguyen

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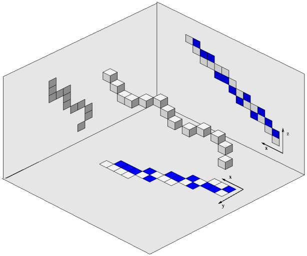
- Linear algorithm of recognition
  - Algorithm of segmentation of a 3D discrete curve into 3D blurred segments for a given width



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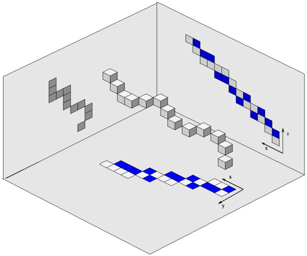
  - Length
  - Curvature



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Curvature and Torsion Estimators for 3D Curves, LNCS , ISVC 2008.

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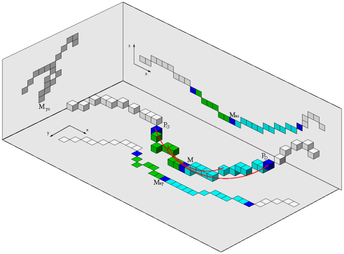

Curvature radius of width 1 and 2 at the point M.

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Outline of talk

Conclusion

Study of regular discrete structures

- Arithmetical, geometrical and combinatorial properties

$\Rightarrow$ Efficient algorithms

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- Other works (with L. Provot) : Discrete planes, Blurred pieces of discrete planes, Segmentation of 3D objects, Geometrical parameters on 3D objects, ...